Chapter 2 Review Questions

1. Suppose that instead of a lump sum tax the government introduced a proportional income tax such that:

$T = tY$ *where t is the marginal tax rate.*

a. What is the new relationship between income and disposable income?

Assume consumption is still a function of disposable income:

$C = a + cY^d$

Where *a* is autonomous consumption and *c* the marginal propensity to consume out of disposable income.

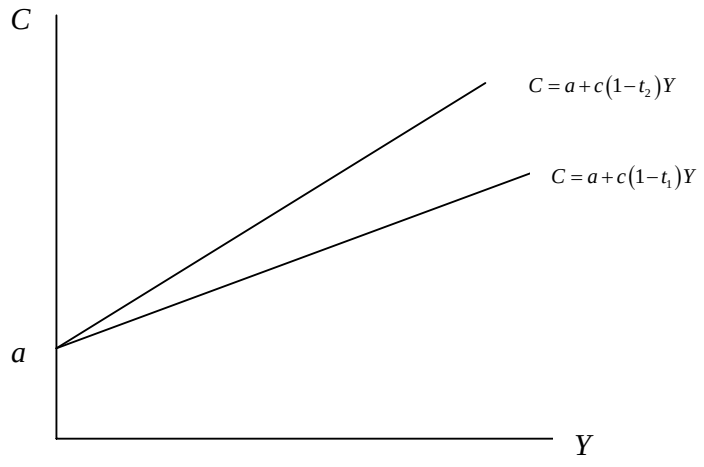
Disposable income is the income left after tax. If the tax rate is *t*, then $Y^d = (1-t)Y$.

Therefore the new consumption function is:

$C = a + c(1-t)Y$

b. Draw the new consumption function and describe what happens when the tax rate changes. How will changes in the marginal tax rate affect the savings ratio (S/Y)?

Consumption is still a linear function of income. The intercept is given by the level of autonomous consumption *a*, and the gradient of the consumption function by $c(1-t)$.



Changes in the tax rate *t* alter the slope of the consumption function but not the intercept- so lead to pivots rather than shifts in the consumption function.

A reduction in the tax rate from $t_1 \to t_2$ increases the level of disposable income at each positive income level. As the marginal propensity to consume is left unchanged consumption rises at each income level and the consumption function pivots upwards.

A pivot results because the effect of the tax change on disposable incomes is proportional to the level of income. When income is zero, changes in *t* have no impact on disposable income so the intercept point remains unchanged. At high levels of income a fall in the tax rate will increase disposable income more substantially.

Household saving refers to disposable income which isn't consumed. The marginal propensity to save is simply $s = 1 - c$, hence the saving function is:

$$S = -a + s(1-t)Y$$

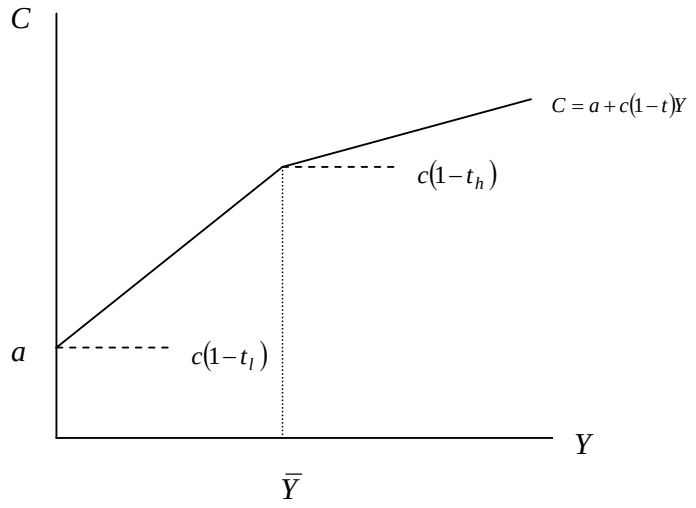
Therefore, the savings ratio (or the average propensity to save) is:

$$\frac{S}{Y} = -\frac{a}{Y} + s(1-t)$$

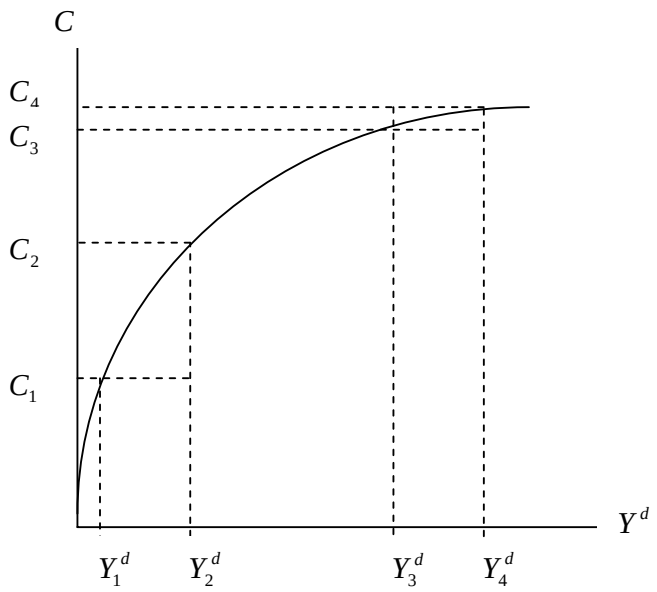
As the tax rate *t* rises the savings ratio falls. Higher proportional taxes reduce the resources available for saving at each level of income. Incidentally, the ratio of saving to disposable income is independent of the tax rate, i.e. $S/(1-t)Y = a/Y + s$.

c. The government decides that a uniform tax rate is unfair. Incomes below $\bar{Y}$ are to be taxed at the rate t_l and incomes above are to be taxed at the higher rate t_h, what effect will this have on the shape of the household consumption function? How might the progressiveness of the tax system affect the level of aggregate consumption?

The progressive tax system introduces a kink to the aggregate consumption function at $\bar{Y}$.



Changing the progressiveness of the income tax system may have implications for the level of consumption, even if the change is neutral in terms of tax revenues. It is commonly argued that high income households have a lower propensity to consume than poorer households- this is a result of the law of diminishing marginal utility of income. As a result the marginal propensity to consume falls with disposable income, giving a concave consumption function.



Changes in disposable income, when combined with different marginal propensities to consume can lead to different levels of consumption. If taxes are reduced on poor household then disposable income rises from Y_1^d to Y_2^d and consumption increases by C_1 to C_2. On the other hand, if taxes are simultaneous increased on rich households disposable income will fall from Y_4^d to Y_3^d, in which case consumption will fall from C_4 to C_3. Although the change in the tax system has left total disposable income unchanged, total consumption has increased because $C_2 - C_1 > C_4 - C_3$.

This example implies that if the marginal propensity to consume falls with income, a more progressive tax system will generate greater consumption, but a less progressive tax system will expand total saving.

2. A household with a life expectancy of five years expects to receive the following income stream at the end of each year.

Year	*Income*
1	*£20,000*
2	*£25,000*
3	*£25,000*

4 *£30,000*
5 *£40,000*

The prevailing interest rate is 10%.

a. What is the expected present discounted value of the household's income?

The general rule for calculating expected present discounted value:

$$EPDV = \frac{E[Y_1]}{(1+r_1)} + \frac{E[Y_2]}{(1+r_1)(1+r_2)} + \frac{E[Y_3]}{(1+r_1)(1+r_2)(1+r_3)} + ...$$

The first period income is discounted because it is received at the end of the year.

Expected present discounted value of household income:

$$= \left(\frac{£20,000}{1.1}\right) + \left(\frac{£25,000}{1.1^2}\right) + \left(\frac{£25,000}{1.1^3}\right) + \left(\frac{£30,000}{1.1^4}\right) + \left(\frac{£40,000}{1.1^5}\right) = £102,953.10$$

b. The interest rate falls to 5%, what is the new present discounted value of the household's income?

$$= \left(\frac{£20,000}{1.05}\right) + \left(\frac{£25,000}{1.05^2}\right) + \left(\frac{£25,000}{1.05^3}\right) + \left(\frac{£30,000}{1.05^4}\right) + \left(\frac{£40,000}{1.05^5}\right) = £119,341.42$$

For the same income flow, a reduction in interest rates acts to increase its present discounted value because future income flows are discounted less heavily.

c. Interest rate starts off at 10%, but after two years falls to 5%. Then, during the final year the interest rate rises sharply to 15%. Recalculate the expected present discounted value of the income stream with this path of interest rates.

$$= \left(\frac{£20,000}{1.1}\right) + \left(\frac{£25,000}{1.1^2}\right) + \left(\frac{£25,000}{1.1^2 \times 1.05}\right) + \left(\frac{£30,000}{1.1^2 \times 1.05^2}\right) + \left(\frac{£40,000}{1.1^2 \times 1.05^2 \times 1.15}\right) = £107,082.03$$

3. Using the optimal consumption model, explain what effect the following will have on current and future consumption:

a. A severe recession is expected with 100% certainty

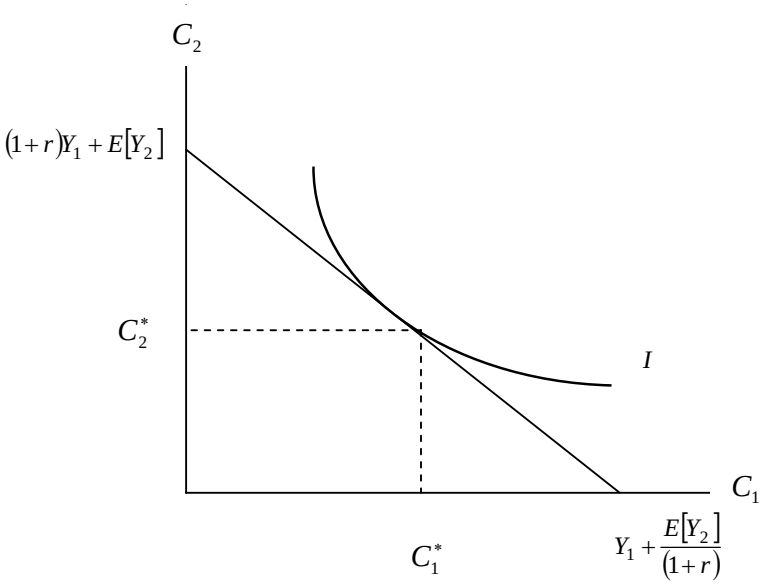
Current consumption is C_1, future consumption is C_2

Expected income in each period is Y_1 and $E[Y_2]$, the interest rate is r.

Therefore, intertemporal budget constraint is such that the discounted sum of consumption is equal or less than total discounted lifetime resources.

$$C_1 + \frac{C_2}{(1+r)} \leq Y_1 + \frac{E[Y_2]}{(1+r)}$$

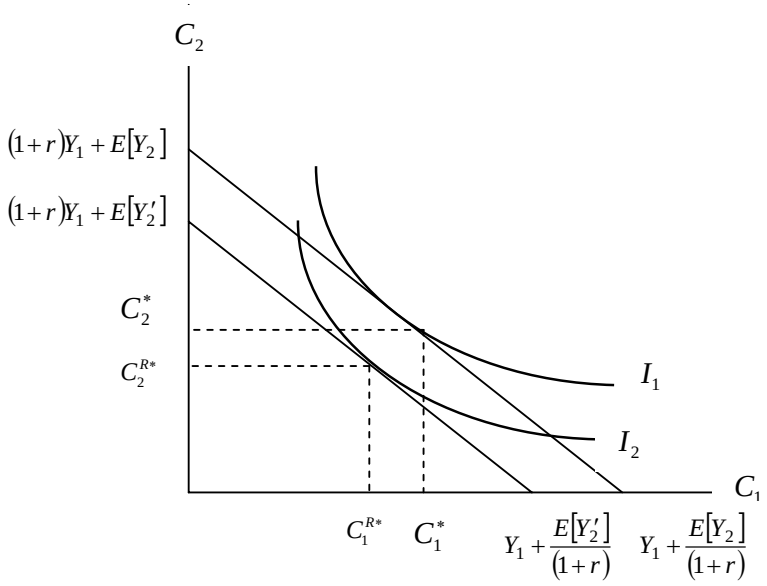
Optimal consumption (C_1^*, C_2^*) is found by maximising utility subject to the budget constraint. This is where the indifference curve forms a tangent to the budget constraint.



A severe recession is expected with 100% certainty:

Therefore, $E[Y_2'] = Y_2^R < E[Y_2]$

Although it is only second period income that is expected to be lower, optimal household consumption falls in both periods. This reflects the household's attempt to maximise lifetime utility by smoothing the path of consumption.



b. A severe recession is expected with a small likelihood of 10%

If the recession is expected with a probability of 10% then

$$E[Y_2''] = 0.9 * Y_2^{NR} + 0.1 * Y_2^{R}$$

Therefore, it should be the case that:

$$E[Y_2'] < E[Y_2''] < E[Y_2]$$

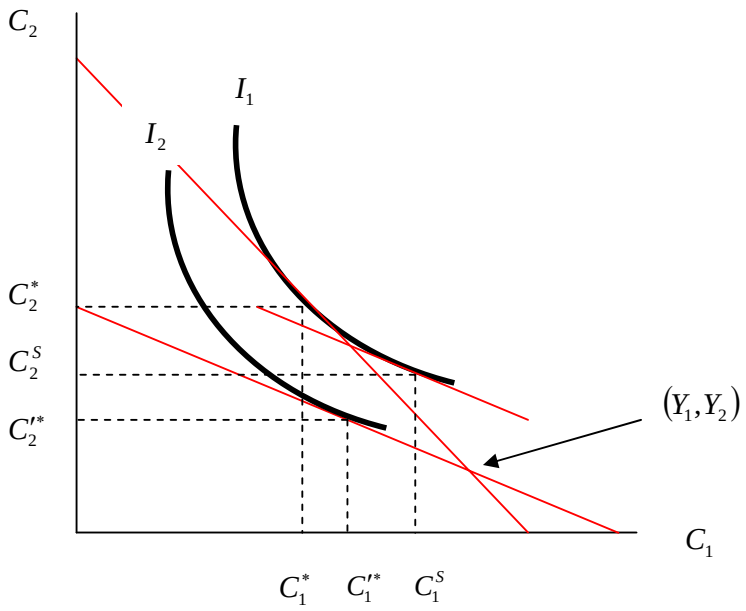
This would have a similar effect as in part a. Lower expected period 2 income will shift the intertemporal budget constraint inwards (but by a smaller amount than in a.) and optimal consumption will fall in both periods (but also by a smaller amount than in part a.).

However, there is a further consideration. In part a. it was known with certainty that a recession will occur. Uncertainty though can generate precautionary saving. In this example there is a small chance of a bad outcome occurring, but if households are sufficiently risk averse they may well save more in period 1 just in case a recession does occur in period 2. Therefore, the fall in period 1 consumption could be larger than implied above.

c. A cut in interest rates

A cut in interest rates makes borrowing more affordable, but also reduces the return to saving. The impact on present consumption depends on whether the household is initially a net-saver or a net-borrower.

A net-saver



A net saver is likely to have high current relative to future income. The household's initial optimal consumption plan is (C_1^*, C_2^*) with net saving equal to $Y_1 - C_1^*$.

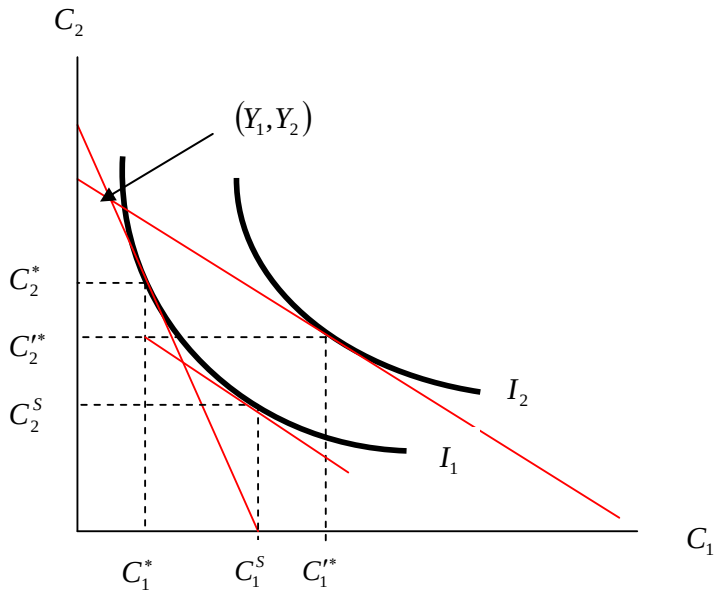
A fall in the interest rate leads to a pivot in the budget constraint through point (Y_1, Y_2).

Maximising utility subject to the new budget constraint sees the household move onto a lower indifference curve $(I_1 \to I_2)$ and choose consumption path $(C_1'^*, C_2'^*)$. Therefore, a net-saver is made worse off following a cut in interest rates.

Lower interest rates reduce the cost of period 1 consumption relative to period 2 consumption which generates a substitution effect $(C_1^* \to C_1^S)$ and $(C_2^* \to C_2^S)$. However, the existing stock of saving now generates lower interest which has a negative income effect and reduces consumption in both periods.

Period 2 consumption will always fall, because the income and substitution effects work in the same direction. Period 1 consumption however can rise or fall depending on which effect dominates.

A net borrower



A net borrower is likely to have low current relative to future income. The household's initial optimal consumption plan is (C_1^*, C_2^*) with net borrowing equal to $C_1^* - Y_1$.

A fall in the interest rate leads to a pivot in the budget constraint through point (Y_1, Y_2).

Maximising utility subject to the new budget constraint sees the household move onto a higher indifference curve $(I_1 \rightarrow I_2)$ and choose consumption path $(C_1'^*, C_2'^*)$.
Therefore, a net-borrower is made better off following a cut in interest rates.

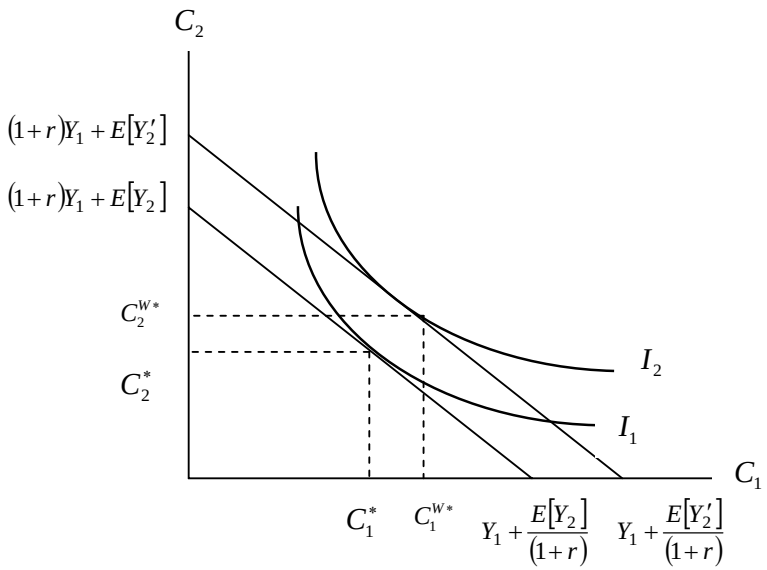
Lower interest rates reduce the cost of period 1 consumption relative to period 2 consumption which generates a substitution effect $(C_1^* \rightarrow C_1^S)$ and $(C_2^* \rightarrow C_2^S)$.
However, the existing stock of borrowing is now financed at a lower interest rate generating a positive income effect that increases consumption in both periods.

Period 1 consumption will always rise in this situation as the income and substitution effects work in the same direction. Period 2 consumption however can rise or fall depending on which effect dominates.

d. An announcement that building societies intend to convert to banks and make windfall payments to depositors

An announcement of cash windfalls from building societies
Expected period 2 income will now rise to reflect windfall payments.

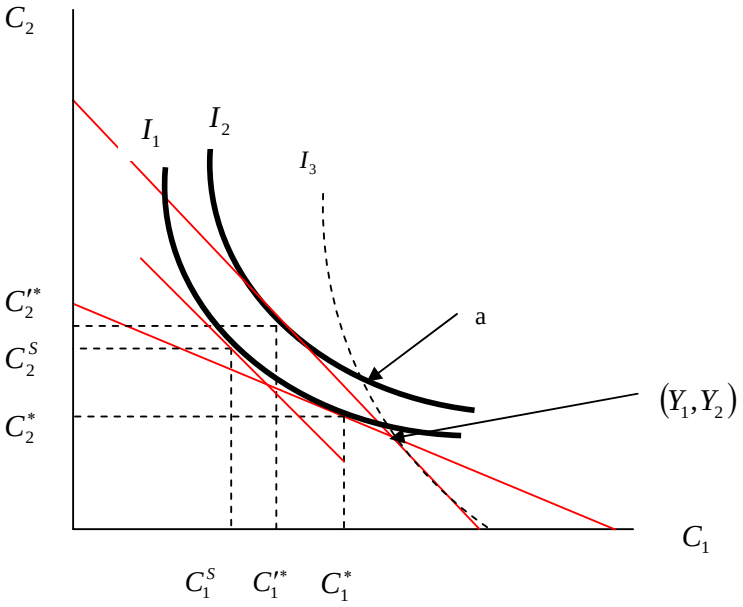
$E[Y_2'] = E[Y]_2 + Y_2^W \quad E[Y_2'] = E[Y_2] + Y_2^W > E[Y_2]$



Higher expected future incomes will shift the intertemporal budget constraint outwards. Maximising utility subject to the new constraint will see the household move onto a higher indifference curve and choose consumption path $\left(C_1^{W*}, C_2^{W*}\right)$. Consumption smoothing behaviour sees current consumption increase even though windfalls are only realised in the future.

4. Explain why a net-borrower might become a net-lender, but a net-lender will never become a net-borrower following a rise in interest rates.

A net saver can never become a net-borrower following a rise in interest rates.



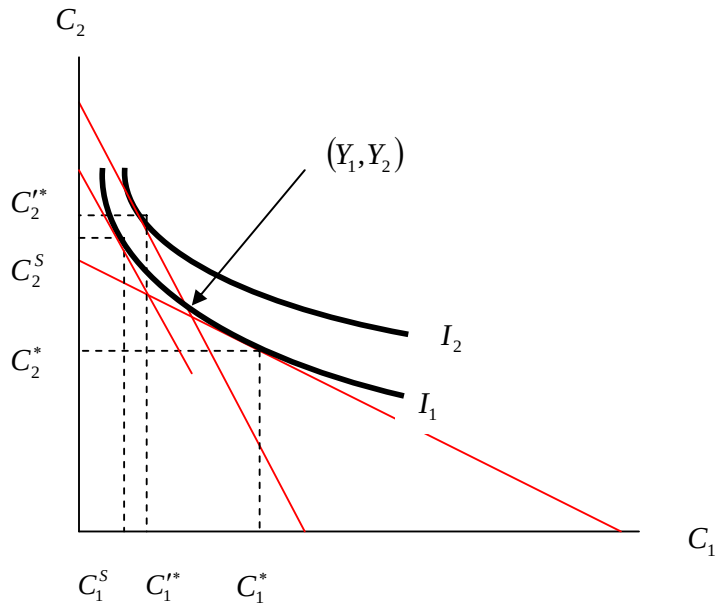
A net saver will never become a net-borrower providing preferences remain consistent.

Starting at the optimal consumption bundle (C_1^*, C_2^*) a rise in interest rates generates a substitution effect towards period two consumption and a positive income effect leading to the new optimal consumption pattern $(C_1'^*, C_2'^*)$. The positive income effect makes the net saver better off as shown in the movement to the higher indifference curve $I_1 \to I_2$.

To become a net saver the point of tangency between the indifference curve and the new intertemporal budget constraint would have to lie to the right of the original income point (Y_1, Y_2). This would require moving to an indifference curve $I_1 \to I_3$ instead of $I_1 \to I_2$.

This however is not possible if preferences are consistent as it would require I_1 and I_3 to cross. The indifference curve marks all the combinations of consumption that give the same level of lifetime utility- so by implication the household should be indifferent between all points on both curves. This is easy to refute, point *a* on I_3 entails higher consumption in both periods than (C_1^*, C_2^*) on I_1 so must be preferable. This inconsistency explains why, if preferences are well-behaved, then indifference curves cannot cross.

A net borrower can become a net saver following a rise in interest rates.



A household's initial utility maximising position is at (C_1^*, C_2^*). This household is a net borrower as $C_1^* > Y_1$.

An increase in interest rates pivots the intertemporal budget constraint through (Y_1, Y_2) leading to the new utility maximising consumption pattern $\left(C_1'^*, C_2'^*\right)$. The household is now a net saver because $C_1'^* < Y_1$.

The substitution effect increases the price of current relative to future consumption, and encourages the household to change consumption patterns accordingly. Providing the substitution effect is strong enough, a net borrower can be turned into a net saver.

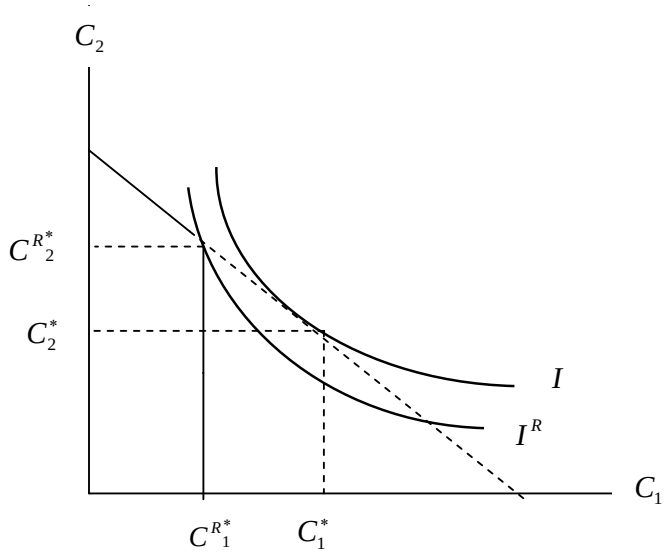
5. Why might spending on durable goods be more closely related to current income than spending on non-durable goods and services?

Non-durable goods and services tend to be consumed at or close to the time of purchase. They can therefore be purchased in small quantities and consumed immediately.

A durable good on the other hand provides a stream of consumption services to the household over its lifetime. Therefore, buying a durable good involves paying upfront for the services it will yield in the future.

Following an increase in expected future income a household may rationally choose to increase the consumption of all types of goods and services. As services and non-durables can be consumed in divisible amounts the cost of increasing current consumption on these items is relatively low. For durables this may not be the case as to increase current consumption we also have to pay to increase future consumption.

If households can borrow and save freely then this presents few problems, as durable goods can be purchased by borrowing against future income. However, if a household were credit constrained then they would have to wait for the increase in income to be realised before the durable good is affordable.



A credit constrained consumer would optimally consume at $\left(C_1^*, C_2^*\right)$, but borrowing constraints mean that the dashed segment of the intertemporal budget constraint is unattainable restricting the consumption path to $\left(C_1^{R*}, C_2^{R*}\right)$. The pattern of consumption is then tied to the pattern of income.

More advanced problems

6. Suppose a household's preferences over consumption in two periods can be represented in a utility function of the following form:

$$U\left(C_1, C_2\right) = u\left(C_1\right) + \rho u\left(C_2\right)$$

a. What interpretation can be given to the parameter ρ ? If the law of diminishing marginal utility of consumption holds how will the parameter ρ affect the shape of the household's indifference curves?

The parameter ρ is the relative weight attached to the utility derived from consumption in each period. The slope of the indifference curve reflects the ratio of the marginal utility of consumption in each period which are $\frac{\Delta U}{\Delta C_1} = MU_{C_1}$ and $\frac{\Delta U}{\Delta C_2} = \rho MU_{C_2}$ respectively.

Hence: $\Delta U = MU_{C_1}\Delta C_1 + \rho MU_{C_2}\Delta C_2$, the total change in lifetime utility is found by the weighted sum of the product of the change in consumption in each period by the marginal utility gained.

Slope of the indifference curve is given where $\Delta U = 0$ because the change in utility along an indifference curve is zero.

$$0 = MU_{C_1}\Delta C_1 + \rho MU_{C_2}\Delta C_2$$

$$\frac{\Delta C_2}{\Delta C_1} - \frac{MU_{C_1}}{\rho MU_{C_2}}$$

As the value of ρ increases, the slope of the indifference curve falls.

The intuitive explanation is that as a higher weight is placed on period two consumption, the household becomes more prepared to trade off a higher amount of period 1 consumption in order to maintain the level of period two consumption.

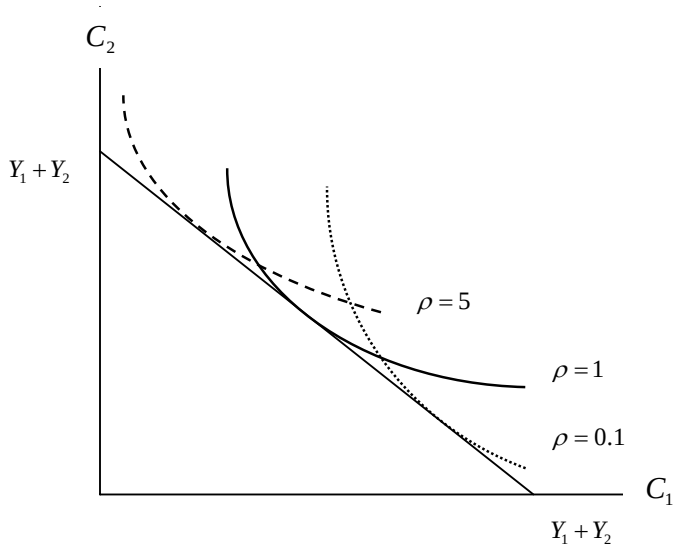
b. A household has income in each period of Y_1 *and* Y_2*, and faces an interest rate of zero? Show the intertemporal pattern of consumption when:*

$\rho = 0.1$

$\rho = 1$

$\rho = 5$

Following the derivation in part a, as the parameter ρ increases higher weight is placed on period 2 utility. These preferences are reflected in the slope of the indifference curve, which becomes flatter implying that the household is more willing to reduce period 1 consumption in order to maintain period 2 consumption. By consequence, the pattern of intertemporal consumption becomes more skewed to the future.



c. Does the inadequacy of personal saving for retirement undermine the personal income and life cycle hypotheses?

The personal income and life cycle hypotheses suggest that individuals smooth consumption over time. Underlying this is the optimal consumption model, where the smoothing results from the convexity of indifference curves due to the law of the diminishing marginal utility of consumption.

However, low current saving may reflect preferences for high current consumption. These preferences though may be the result of myopia- and that households' short sightedness may be detrimental to their long-term welfare. In this scenario policy may

be directed at influencing the value of ρ. If saving for retirement is an adequate households may be encouraged to save more in pension funds if the value of ρ can be raised.

7. A household receives income in period 1 of Y_1 and income in period 2 of Y_2. A bank though charges a different interest rate on savings and (r_s) loans (r_l).

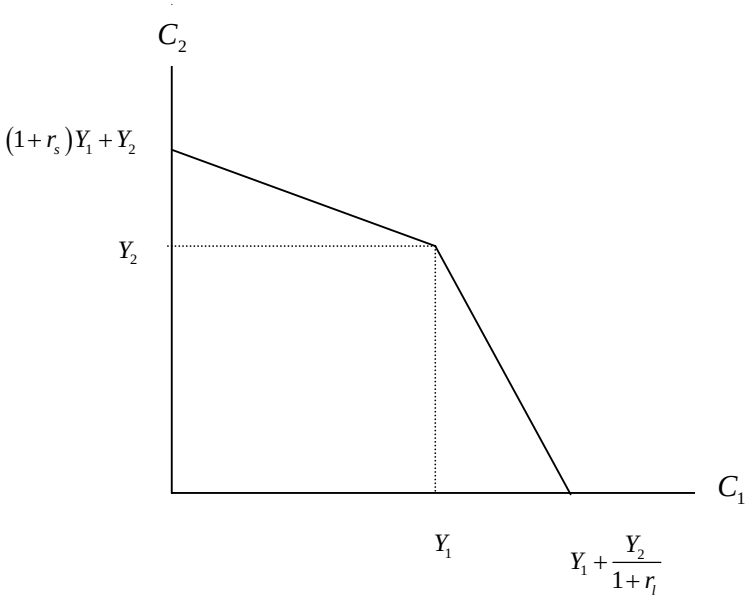
a. Why might a bank charge a higher rate of interest on loans than it pays on savings accounts?

There are two main reasons for a spread between borrowing and lending rates of interest.

Profit: Funds deposited in a financial institution can be loaned out. If the interest on loans exceeds that on savings then positive profits are made. The size of the spread may reflect the degree of competition for both saving and loans, with a larger spread consistent with lower competition.

Risk: Financial institutions do not have perfect information concerning the investments their loans will be used to fund. There is always a chance that the borrower may go bankrupt and default on their liabilities. Therefore, a risk premium to cover this is attached increasing the borrowing rate relative to the saving rate.

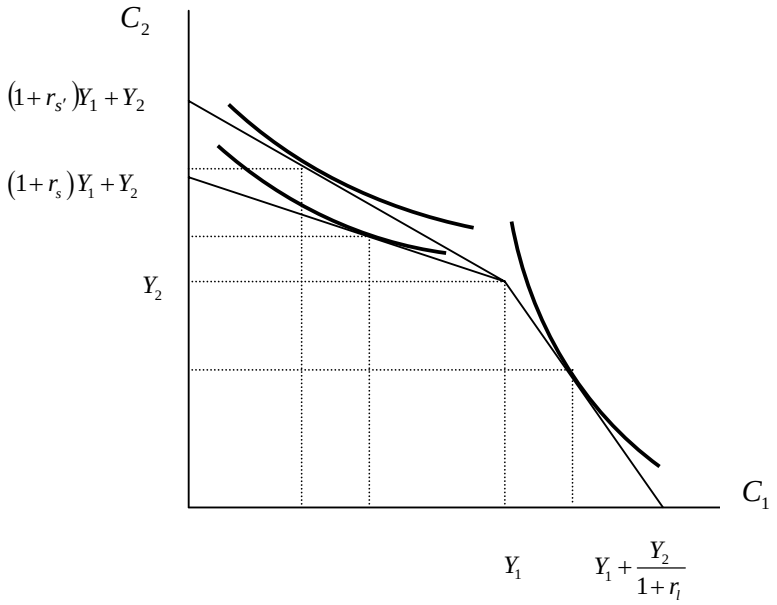
b. Draw the intertemporal budget constraint when $r_l > r_s$.



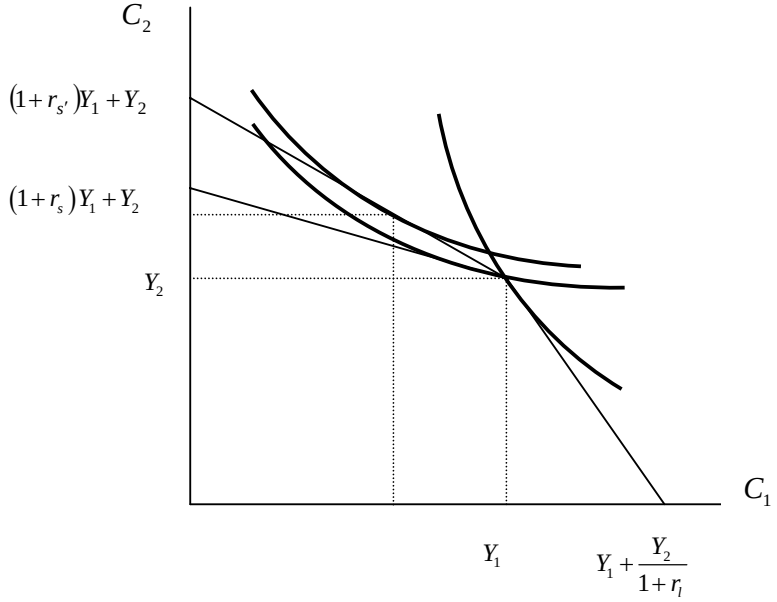
When borrowing rates exceed the saving rates the intertemporal budget constraint develops a kink at the point where there is neither saving nor borrowing.

c. In an attempt to attract new savers the bank raises the interest rate on saving accounts to r_s' so that $r_s < r_s' < r_l$. What will the new intertemporal budget constraint look like? What effect will this have on the intertemporal consumption pattern of households? Will it depend on whether the household was initially a net-saver or a net-borrower?

Following an increase in the saving interest rate r_s to $r_{s'}$ the intertemporal budget constraint pivots through the current income position (Y_1, Y_2) but remains kinked.



From the above diagram higher interest rates encourage existing savers to save more, but do not convert net borrowers into net savers. However, as shown below, those who are neither net-borrowers nor net savers may be encouraged to become net savers.



In this situation, those who are likely to be persuaded to become net-savers are those with flatter indifference curves. These are households who are more readily willing to swap period one for period two consumption, so interest rate changes have strong substitution effects. These households are also more likely to be net-savers in the first place, as the top diagram shows.

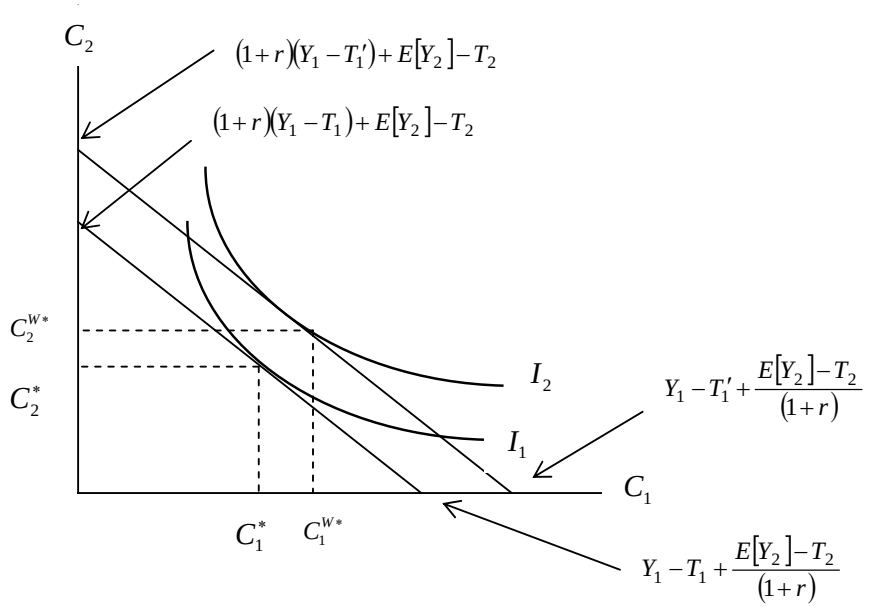
8. As an election approaches the government wishes to stimulate the economy by boosting current consumption. In order to achieve this it has proposed an immediate and one-off tax cut. Explain how the following factors might affect the success of the policy?

a. Households are permanent income consumers

If households are permanent income consumers then the one-off tax cut will only increase consumption to the extent it raises permanent income. Therefore the tax cut will be spread over the entire lifetime. The intertemporal budget constraint is:

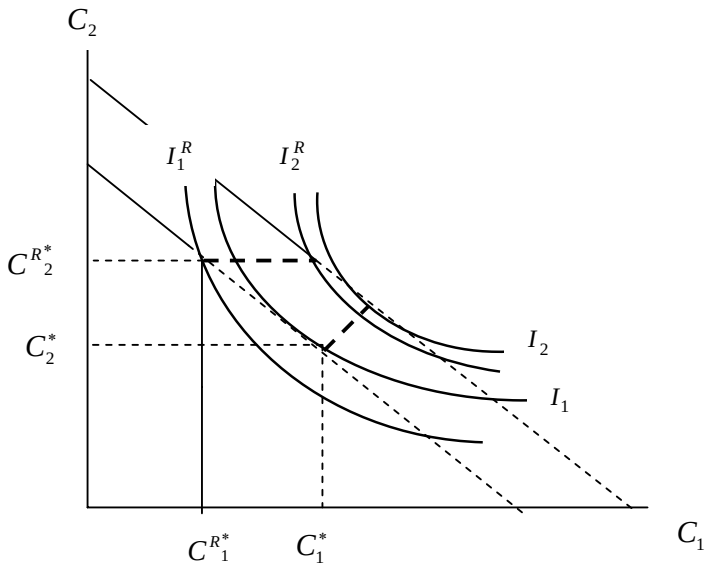
$$C_1 + \frac{C_2}{(1+r)} \leq Y_1 - T_1 + \frac{E[Y_2] - T_2}{(1+r)}$$

A fall in current taxes, will raise lifetime disposable income.



The tax cut will boost current consumption. However, the increase in current consumption will relatively small relative to the change in disposable income, as part of the tax cut is saved to boost future consumption.

b. The economy consists of a large number of credit constrained households

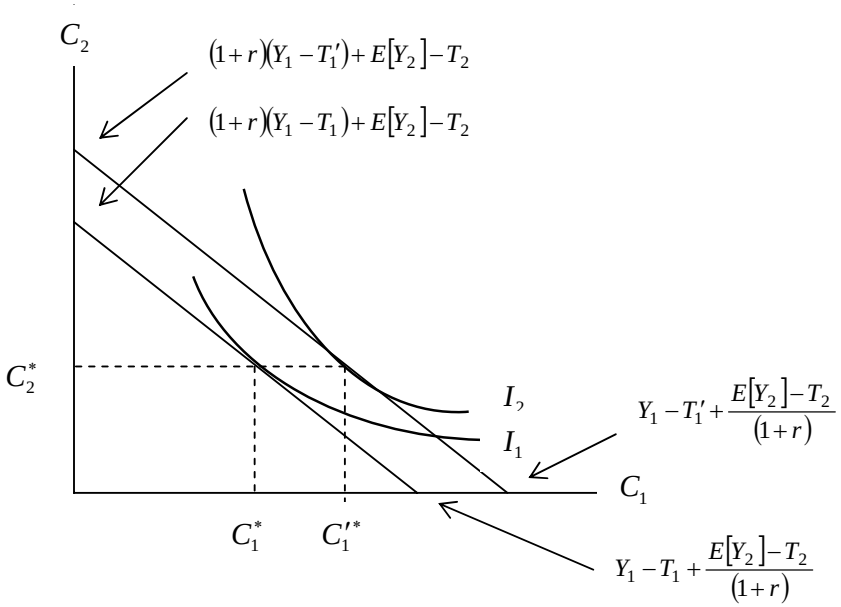


The impact of the tax cut has a much larger effect on current consumption if initially there are credit-constrained households. In this diagram the dashed segment of the budget constraints represents that which can only be achieved by borrowing against future income. If this is not possible then these consumption bundles are unattainable.

For an unconstrained household, the tax cut will have the same impact as in part a. However, for the constrained household the rise in current consumption would be much greater. In fact, in the example above consumption will increase by the full extent of the increase in disposable income.

c. The tax cut produces a feel good factor leading to a change in preferences towards current consumption

If the tax cut also leads to a change in preferences towards current consumption then the increase in current consumption will be greater than in part a. Here the indifference curves become steeper representing the households increased preference for current consumption. The extent to which the increase in disposable income is smoothed over the lifetime is then reduced.



d. The general public are aware that the government has financed the tax cut through borrowing- which must be repaid with interest after the election

Under the optimal consumption model current consumption will only change if there is a shift or pivot in the intertemporal budget constraint. If the government borrows to fund the tax cut then the budget deficit rises by $-(\Delta T)$, where a tax cut implies $\Delta T < 0$. In the future though the government must repay this borrowing with interest, so future taxes must rise by $(1+r)(-\Delta T)$.

In present value terms, the household's budget constraint will therefore change by:

$$-\Delta T - \frac{(1+r)(-\Delta T)}{(1+r)} = 0$$

Hence, there is no change in the present discounted value of disposable income so no change in consumption would be expected. This forms what is known as 'Ricardian Equivalence' which is explained in more depth in chapter 4.

9. Different theories of consumption behaviour mainly reflects different assumptions about the quality of capital markets"- Discuss

This is concerned with the link between consumption and current income. The Keynesian consumption function posits a direct linear association between the two. The permanent income hypothesis though breaks the link between consumption and

current income, by suggesting that consumption plans are based on a long term view of average, or permanent, income.

One of the strong implications of the permanent income hypothesis is that consumption follows a unit root with drift (note that the full assumptions for this are described in R. Hall (1978), *Stochastic implications of the life cycle permanent income hypothesis: theory and evidence*, Journal of Political Economy 971-987).

$$\Delta C_t = \mu + \varepsilon_t$$

This suggests that the change in consumption is equal to a constant and a random error term. Plotting a unit root with drift process would see the series moving randomly around a trend line. The main intuition is that changes in consumption should be unpredictable. If households are fully optimising, then the only thing that should lead to a change in consumption patterns should be unpredictable income shocks. If a change in income is predictable it should have no effect on current consumption because its effect should already have been incorporated into the choice of consumption patterns.

This can be tested by running the following regression:

$$\Delta C_t = \mu + \beta \Delta Y_t + \varepsilon_t$$

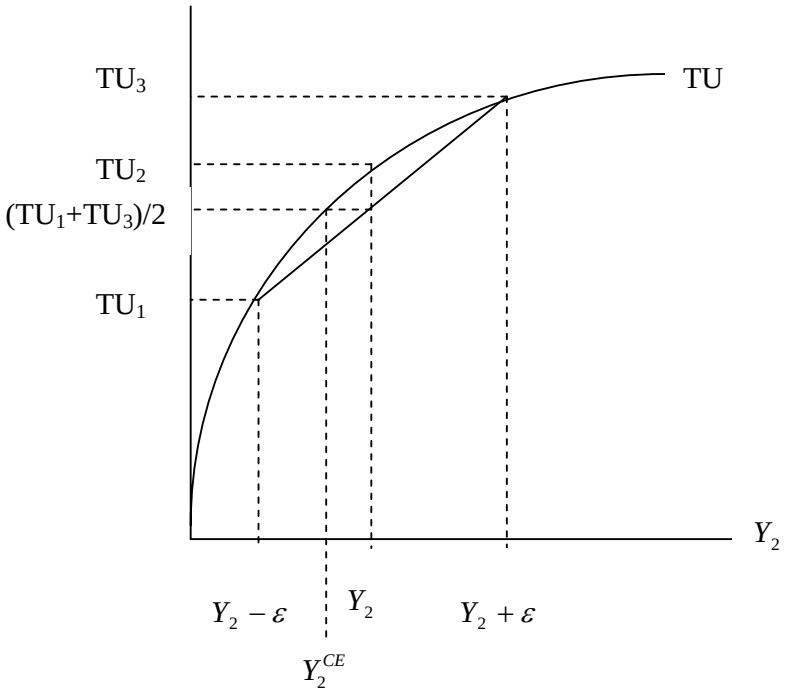
If β is significantly different from zero then it implies a divergence from the PIH. Empirical evidence tends to support this and is known as the excess sensitivity hypothesis. The main explanation is that households face borrowing constraints, which ties current consumption to current income.

10. Explain why households pay insurance premiums. What effect might a reduction in social security payments have on the saving ratio?

The cost of insurance represents the price for dealing with uncertainty. Suppose future income is uncertain $Y_2 \pm \varepsilon$, where $E[\varepsilon] = 0$

Given the choice between having Y_2 with certainty, or a gamble between $Y_2 + \varepsilon$ and $Y_2 - \varepsilon$ with 50% chance each, the household will almost certainly opt for the sure amount. This is because although the gamble represents a fair bet in money terms, it does not in utility terms.

The law of diminishing marginal utility of income/consumption implies that the total utility function is concave. Therefore, the certainty equivalent of the gamble is lower than the average value of the gamble. The household would then be prepared to pay up to the difference $Y_2 - Y_2^{CE}$ to avoid the uncertainty altogether.



A reduction in social security would be expected to increase the variance of disposable income. With no social security, income tax would be lower so disposable income is higher if working. If though unemployment or incapacity removes you from the labour market, there are fewer benefits so income is lower.

Without the social security safety net future disposable income is less certain. Therefore, a household may be expected to increase current saving to cover the risk of low future income should the worse happen. Precautionary saving motives leads to an increase in the saving ratio.

What conditions are required for a precautionary saving motive?

The utility maximising position for a two period optimiser is found where the indifference curve forms a tangent to the budget constraint. The slope of the indifference curve reflects the ratio of marginal utility of consumption in each period, and the slope of the budget constraint equals the interest rate as this is the cost of transferring wealth over time. Therefore, the optimising position requires:

$$-\frac{MU_{C_1}}{MU_{C_2}} = -(1+r) \text{ implying } MU_{C_1} = (1+r)MU_{C_2}.$$

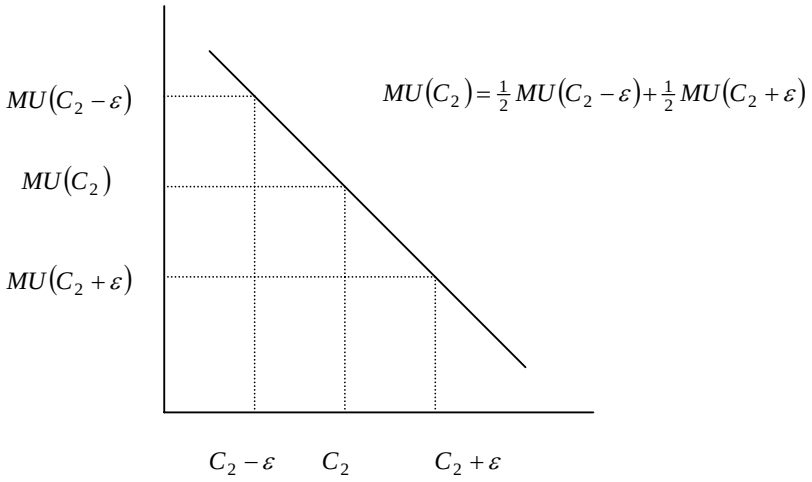
If $MU_{C_1} < (1+r)MU_{C_2}$ then it implies that the household can be made better off by transferring consumption from period 1 to period 2.

Because future income is uncertain then this equality can be expressed in terms of expectations:

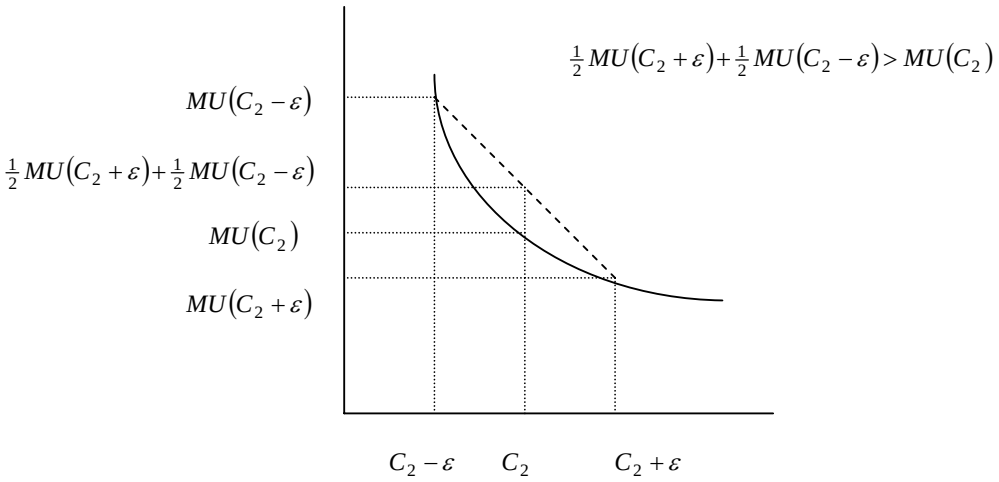
$$MU_{C_1} = (1+r)E\lfloor MU_{C_2} \rfloor$$

A precautionary saving motive arises when the marginal utility function is convex, so the second derivative of the utility function with respect to future consumption is negative.

When utility functions are quadratic, the second derivative will be zero and the marginal utility of consumption is linear. In this case uncertainty over future consumption will have no impact on the calculation of $E\lfloor MU_{C_2} \rfloor$.



If the marginal utility is convex, then uncertainty increases the average of marginal utilities.



Now, the effect of uncertainty means that $E\lfloor MU_{C_2} \rfloor > MU_{E[C_2]}$. Going back to the optimising condition $MU_{C_1} = (1+r)E\lfloor MU_{C_2} \rfloor$ it is clear that increasing the uncertainty of future consumption implies that $MU_{C_1} < (1+r)E\lfloor MU_{C_2} \rfloor$. As a result, utility optimisation requires reallocating consumption from the present to the future. This is achieved by increasing current saving- this is known as precautionary saving because it is solely in response to future uncertainties.