Chapter 3 Review Questions

1. What are the three main components of private investment?

Three main components of investment

Fixed business investment
This represents expenditure by firms on additions to their capital stock. A firm's stock of capital is all the physical assets at its disposal which can be used to produce output, including items such as machinery and computer hardware.

Inventory Investment
Inventories are stocks of inputs, semi-completed and finished goods that firms hold in storage.

Residential Investment
This refers to investment on improving or building residential property.

2. Using the optimal capital stock model, explain how fixed private investment undertaken by a firm might be influenced by the following:

a. An increase in productivity

Firm output is a function of its capital stock and productivity *A*.

$$Y = AF(K)$$

Changes in the parameter *A* mean that the same capital stock can produce different levels of output, so leads to shifts in the firm production function.

The marginal product of capital is the extra output that is derived from an additional unit of installed capital.

$$MPK = \frac{\Delta Y}{\Delta K} = A\frac{\Delta F(K)}{\Delta K}$$

Assuming that the production function exhibits diminishing returns then the marginal product of capital falls as the capital stock rises.

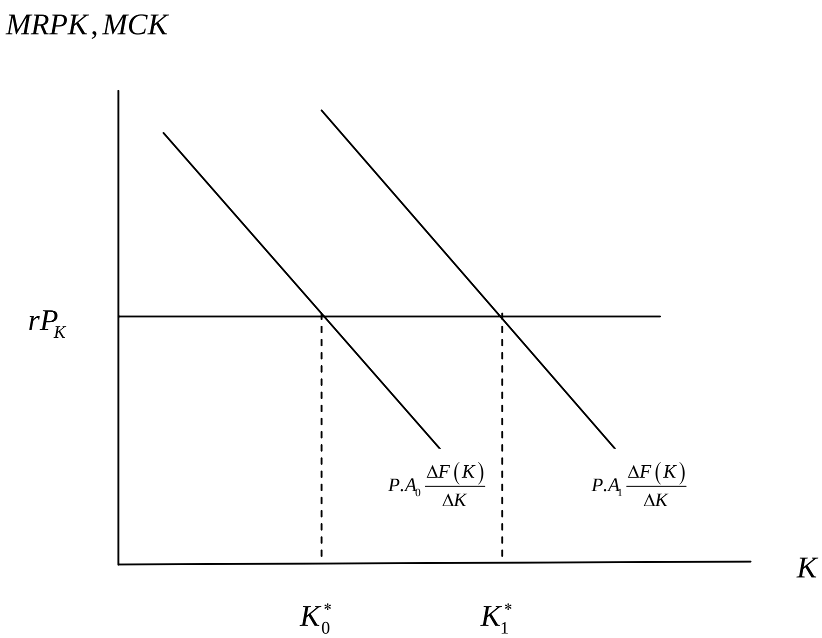
Changes in *A* lead to shifts in the marginal productivity of capital curve- so that a different MPK results at each level of capital stock. The marginal revenue product of capital is equal to the value of the extra output from an additional unit of installed capital, which is simply $MRPK = P.MPK$.

The marginal cost of an extra unit of installed capital is determined by the price of capital goods P_K and the rental rate r, $MCK = rP_K$.

The optimal capital stock is where the marginal revenue product and marginal cost of capital are equal.

$$P.A\frac{\Delta F\left(K\right)}{\Delta K}=rP_K$$

An increase in productivity leads to an increase in the optimal capital stock and thus investment.

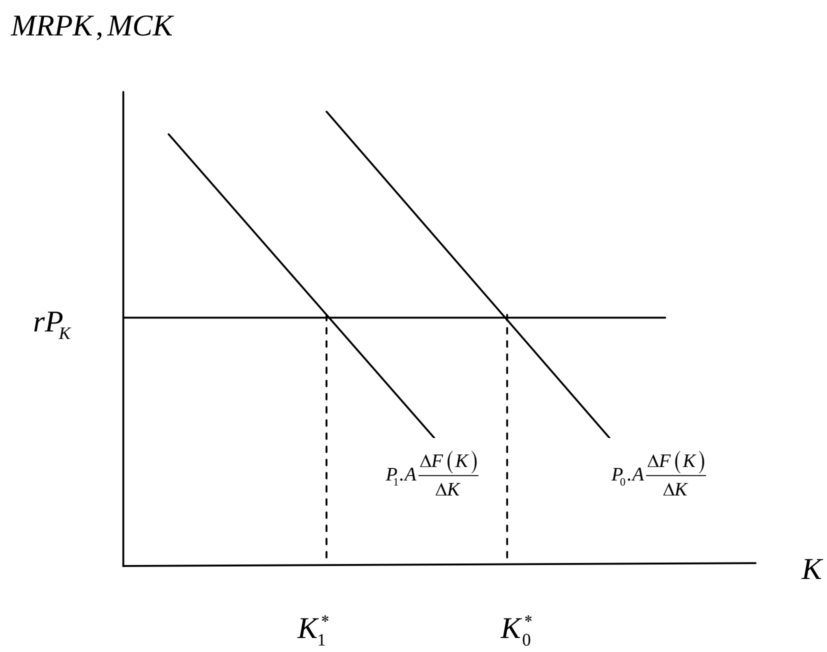


At A_0 the optimal capital stock is K_0^*, whereas at $A_1 > A_0$ the optimal capital stock is K_1^*. Investment thus responds to close the gap between current and optimal capital stocks, $I=\gamma\left(K_1^*-K_0^*\right)$.

b. A fall in the demand for a firm's product

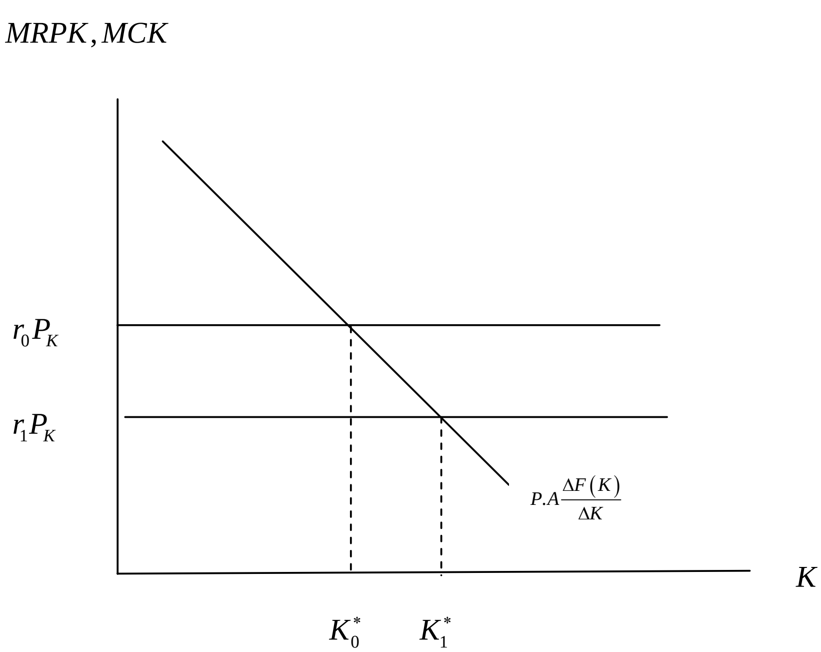
This would be expected to lower the price of the product. Therefore, each extra unit of output generates lower revenue leading to an inward shift in the marginal revenue product of capital.

As prices fall from $P_0 \rightarrow P_1$ the optimal capital stock declines $K_0^* \rightarrow K_1^*$ and investment responds to move the existing capital stock towards its optimal level. $I=\gamma\left(K_1^*-K_0^*\right)$



c. A fall in interest rates

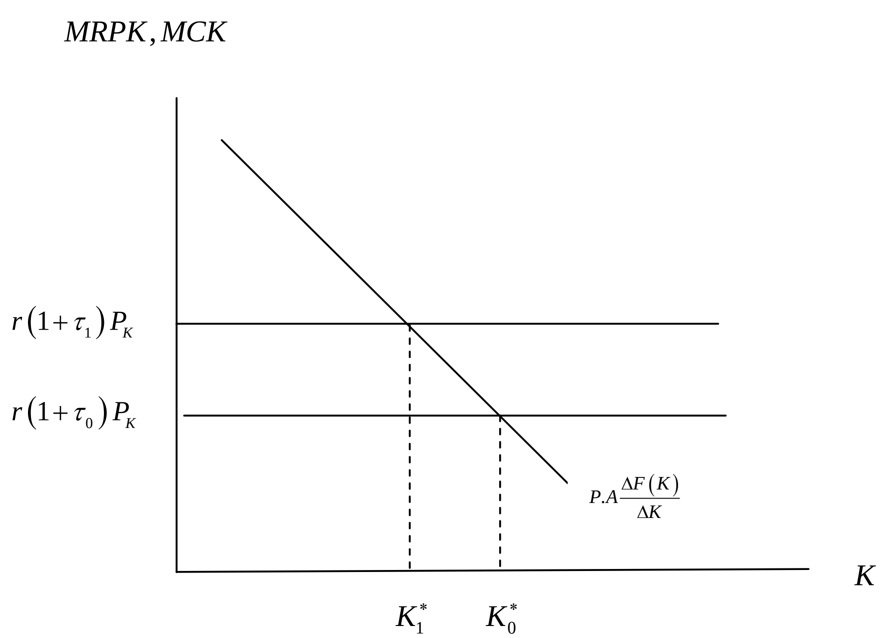
Interest rates are referred to as the opportunity cost of capital, i.e. instead of investing in the capital stock a firm could alternatively purchase a bond and achieve a given rate of interest. A fall in interest rates lowers the marginal cost of capital.



As rates fall from $r_0 \rightarrow r_1$ the optimal capital stock increases by $K_0^* \rightarrow K_1^*$. The marginal product of capital falls as the capital stock rises, hence lower interest rates enable positive profits to be earned on newly installed capital that was unprofitable at the previous level of interest rate. Investment increases in response to the change in the optimal capital stock $I = \gamma\left(K_1^* - K_0^*\right)$.

d. A tax on the purchase of new capital goods

If a tax rate of τ is applied to capital goods then the marginal cost of each extra unit of installed capital is $r(1+\tau)P_K$. An increase in the tax rate thus increases the marginal cost of capital at each level of capital stock.



An increase in the tax rate from $\tau_0 \rightarrow \tau_1$ reduces the optimal capital stock and leads to disinvestment.

e. An increase in the rate of depreciation

The rate of depreciation describes how quickly capital goods wear out. Depreciation alters the cost of capital. Suppliers of capital will charge a rental rate determined by the level of interest rates- but bonds do not wear out so depreciation costs must also be recouped. If the rate of depreciation is δ then the effective marginal cost of capital is $(\delta + r)P_K$.

An increase in the depreciation rate, by raising the marginal cost of capital would have the same effect on the optimal capital stock and investment as described in part d.

3. Using different models of investment, explain why investment might be highly correlated with the economic cycle.

From the optimal capital stock model:

Strong demand can support higher prices. This raises the marginal revenue product of capital and increases the optimal capital stock level as shown in 2b.

Alternatively, higher demand raises output and higher levels of output might make more productive technology cost effective encouraging investment as in 2a.

From the accelerator model of inventories:

The stock of inventories held is proportional to the level of output. Hence inventory investment is governed by changes in output. The parameter v indicates the sensitivity of inventory investment to output movements.

$$K_{inv} = vY$$

$$\Delta K_{inv} = v\Delta Y$$

$$I_{inv} = v\Delta Y$$

Inventory investment is more likely to be sensitive to changes in output when the cost savings of production smoothing are low relative to the costs of storing inventories.

From Tobin's q:

The value of a firm is equal to the discounted value of future cash flows generated by its capital stock.

$$V_0 = \frac{P.F(K_1)}{(1+r)} + \frac{P.F(K_2)}{(1+r)^2} + \frac{P.F(K_3)}{(1+r)^3} + ...$$

The cost of the initially installed capital stock (K_0) is also as before:

$$P_K K_0 .$$

Tobin's q is the ratio of the two.

$$q = \frac{V_0}{P_K K_0}$$

If q exceeds unity then the value of the firm exceeds its replacement cost of capital and investment is positive. This is because there is some intrinsic about the firm that means the increase in value created exceeds the cost of additional capital stock.

$$I = \frac{1}{\theta}(q-1)$$

Similar to the optimal capital stock model, strong demand would be expected to increase the value of future cash flows generated by the firm and hence raise the value of q and investment.

From credit supply model:

Financial institutions have imperfect information concerning the investment projects firms may wish to borrow for and invest in. Bankruptcy rules and limited liability also create incentives for firms to undertake risks. The threat of default limits the availability of credit.

In times of high economic growth credit constraints are likely to be less binding. Firstly, the threat of bankruptcy is lower so banks are less likely to ration credit. Second, firms with higher profitability will have access to larger internal sources of investment funds.

4. A firm faces two potential cash flows connected with a new investment. If the project is successful, the investment will generate high future cash flows in the following years. However, if the project is unsuccessful, then the future cash flows will be low.

a. If the firm believes that the investment project will be successful with probability p (hence unsuccessful with probability 1-p), how will it evaluate the present discounted value of the investment?

A firms cash flow is described by it revenues $\Pi = P.F(K)$.

If the project is successful then high cash flows (Π_H) are obtained each period. The present discounted value of future cash flow is hence:

$$V_H = \frac{\Pi_H}{(1+r)} + \frac{\Pi_H}{(1+r)^2} + \frac{\Pi_H}{(1+r)^3} + ... = \sum_{i=1}^{\infty} \frac{\Pi_H}{(1+r)^i}$$

Likewise, if the project is unsuccessful then low cash flows are obtained each period (Π_L) giving the present discounted value of future flows as:

$$V_L = \frac{\Pi_L}{(1+r)} + \frac{\Pi_L}{(1+r)^2} + \frac{\Pi_L}{(1+r)^3} + ... = \sum_{i=1}^{\infty} \frac{\Pi_L}{(1+r)^i}$$

It is uncertain whether the project will or will not be successful. The expected present discounted value of the project's future cash flows is determined by the relative probabilities of each.

$$E[V] = \rho V_H + (1-\rho)V_L$$

b. Keynes believed that investment was driven by 'animal spirits'- waves of optimism and pessimism. Explain how this model can be used to show Keynes' idea.

The cost of the project is the same at *C*, regardless of success of failure. A q-based evaluation of the project would suggest:

$$q = \frac{E[V]}{C}$$

If $\frac{V_H}{C} > 1$ and $\frac{V_L}{C} < 1$ then the probability of success could be pivotal in whether or not the investment project is undertaken.

But, how are the relative probabilities of success or failure chosen?

If probabilities are formed subjectively by firms then they could be open to influence by external factors.

If firms are optimistic about the future then the probability of success could be high suggesting $q = \frac{E[V]}{C} > 1$ and the investment is undertaken. However, if firms are pessimistic then $q = \frac{E[V]}{C} < 1$ and the project is not undertaken.

Waves of optimism and pessimism can therefore influence investment via the formation of subjective probabilities. These waves can be self-reinforcing, where the beliefs of one firm influence the belief of others leading to herd type movements in investment.

c. Over time, information concerning the likely success of an investment project becomes increasingly available. How might this affect the path investment takes in the economy? Why might it be significant whether or not there are high costs in making investments?

The q theory of investment assumes that managers play no role in the planning and undertaking of investment projects. If a certain investment project has a q value greater than 1, then the firm commits to the investment project. However, this is similar to a poker player making their final bets having been dealt their initial hand. Instead, a poker player may commit just enough to stay in the game at each stage. Each of these payments gives the player an option to continue, or to fold at each point.

According to Dixit and Pindyck (*Investment under uncertainty*, Princeton University Press, 1999) the same intuition applies to investment. Investment projects have managerial options embedded in them, and given uncertainties these options have value. Therefore investment is likely to be a staggered process whereby managers re-evaluate the merits of the project at various stages. This is most likely to be the case where there are high sunk costs and considerably uncertainty regarding the probability of success.

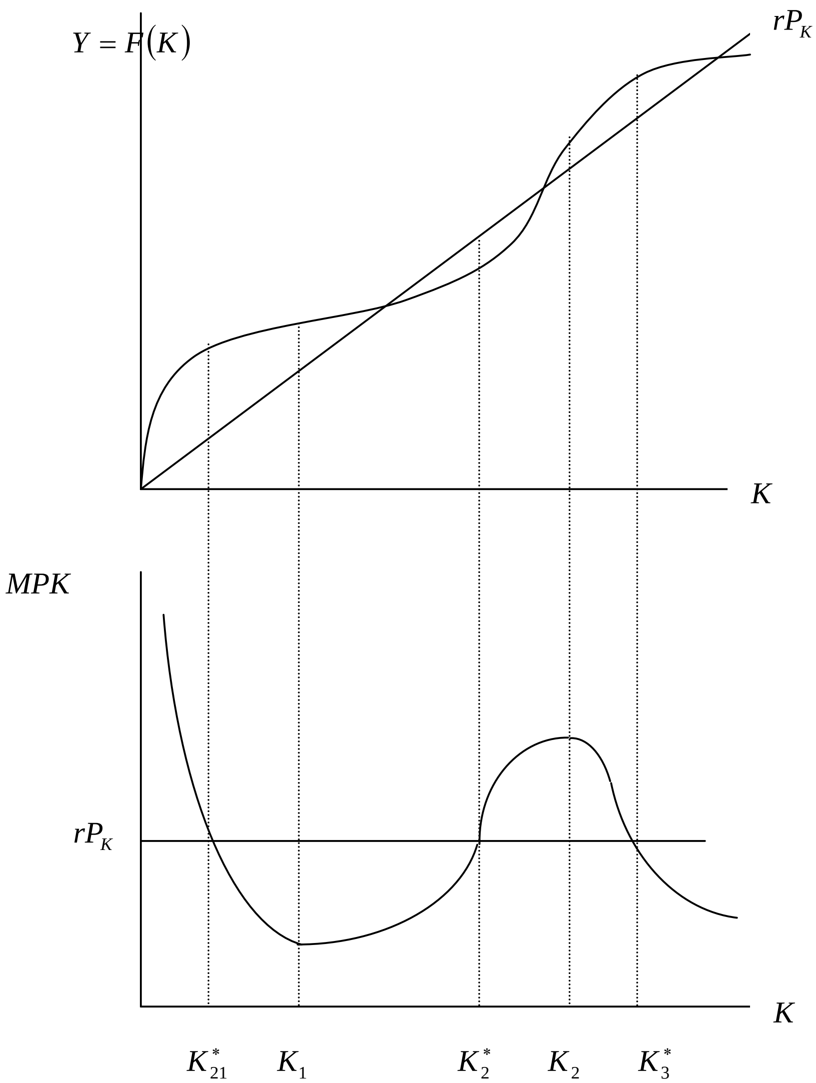
5. A firm's production technology implies that the production function has a zone in which there are increasing returns to capital. Everywhere else the production function exhibits decreasing returns to scale with respect to capital.

a. Sketch both the production function described above and the corresponding marginal productivity of capital schedule.

Increasing returns to scale changes the shape of the firm production function. Where there are decreasing returns to scale the production function is concave. The production function is convex at the points where there are increasing returns.

This is shown in the figure above. Decreasing returns apply in the capital stock ranges $\left[0, K_1\right]$ and $\left[K_2, \infty\right]$, here the marginal product of capital is negative. Increasing returns exist in the range $\left[K_1, K_2\right]$ where the marginal product of capital schedule becomes upward sloping.

Equilibrium investment is where the marginal revenue product and marginal cost of capital are equalised. In this example there are multiple equilibria- but not all of these equilibria are stable.



b. What happens to investment once the capital stock falls within the range of increasing returns to capital?

There are three equilibrium positions where the marginal revenue product and marginal cost of capital are equalised, but only K_1^* and K_3^* are stable, both representing profit maximising positions. These points are reached if the capital stock falls in the ranges $[0, K_1]$ and $[K_2, \infty]$ respectively.

Equilibrium position K_2^* is unstable. Here losses are made and the firm will do better to move away from this position, but will do so regardless of which direction in which it moves. If the firms capital stock was in the range $[K_1, K_2^*]$ marginal costs exceed the marginal revenue product so the firm would reduce investment back to capital stock level K_1^*. Alternatively, if the capital stock is in the range of $[K_2^*, K_2]$ then the marginal revenue product exceeds the marginal cost and the capital stock converges to K_3^*.

c. What explanations might account for a zone of increasing returns to capital in a production function?

A conventional explanation is that once output has passed a certain threshold it is possible to introduce new more productive technology that allows the firm to benefit from scale economies. Murphy, Shleifer and Vishny (1987) (The big push, Chicago Business School- mimeo) investigate at what point an increasing returns technology should be implemented.

6. Tobin's q implies a linkage between firm investment and stock prices. How might the following affect the value of q, stock prices and the level of investment?

a. an increase in the current interest rate paid on bonds

Cash flows generated by in each period are given as $P.F(K_t)$. The value of a firm is equal to its expected discounted future cash flows

$$V_0 = \frac{P.F(K_1)}{(1+r)} + \frac{P.F(K_2)}{(1+r)^2} + \frac{P.F(K_3)}{(1+r)^3} + \text{...........}$$

The cost of the initially installed capital stock (K_0) is also as before:

$$P_K K_0 .$$

Hence

$$q = \frac{V_0}{P_K K_0}$$

If $K = K_0 = K_1 = K_2 = K_3 = ... = K_\infty$ then $V_0 = \dfrac{P \cdot F(K)}{r}$

Hence

$$q = \frac{P.F(K)}{rP_K K}$$

Optimal capital stock is such that $q = 1$, and investment responds positively to the deviation of q to unity.

$$I = \frac{1}{\theta}(q-1)$$

This represents the opportunity cost of capital investment. An increase in the interest rate reduces the present discounted value of future cash flows and therefore lowers the value of q.

b. an expected increase in future interest rates

This will have a similar effect as before. Even though higher interest rates are applied in the future, all subsequent cash flows will be discounted at a higher rate. Consequently, q will fall below 1 and investment will fall.

c. increasing uncertainty about the path the economy will take

Increasing uncertainty

Suppose future cash flows are now given by $P.F(K_t) \pm \varepsilon$ where ε is a fixed sum that is equally likely to be positive or negative. Therefore the PDV is:

$$\tilde{V}_0 = \frac{P.F(K_1) \pm \varepsilon}{(1+r)} + \frac{P.F(K_2) \pm \varepsilon}{(1+r)^2} + \frac{P.F(K_3) \pm \varepsilon}{(1+r)^3} + ...$$

However, given that $E[\varepsilon] = 0$ then the expected present discounted value is $E[\tilde{V}_0] = V_0$.

Therefore, the presence of uncertainty has no effect on the determination of *q* as long as the expected present discounted value of future cash flows remains unchanged.

Is this reasonable? Most firms are relatively risk neutral due to limited liability and bankruptcy procedures. Large firms are also able to diversify risks, so are not as sensitive to general uncertainty as households. Smaller firms may not have limited liability in which case uncertainty is more important.

d. an announcement that a competitor firm is experiencing financial difficulties

This could go either way- a sign of relative success or collective failure. The first implies that competition in the firm's product market is reduced generating higher future profits. However, the second suggests that the demand for that industry's product has fallen and hence cash flows in the future will be lower.

e. a government announcement of lower corporation taxes in the future

A proportional tax ω on the cash flows generated by a firm's capital stock reduces the value of q, because net cash flows are lower. Hence, a reduction in these tax rates would be expected to lead to higher investment.

$$V_0 = (1-\omega)\frac{P.F(K_1)}{(1+r)} + (1-\omega)\frac{P.F(K_2)}{(1+r)^2} + (1-\omega)\frac{P.F(K_3)}{(1+r)^3} + \ldots\ldots\ldots\ldots$$

$$\mathrm{q} = (1-\omega)\frac{P.F(K)}{rP_K K}$$

7. Two different firms each undertake an identical investment project that yields positive profits. Firm 1 intends to distribute these profits as dividend payments to its shareholders, whereas firm 2 decides that the profit stream will be used to fund further investment. How might the share prices of the two firms differ?

The value of a firm is equal to the expected future cash flows that are generated:

$$V_0 = \frac{P.F(K)}{(1+r)} + \frac{P.F(K)}{(1+r)^2} + \frac{P.F(K)}{(1+r)^3} + \ldots\ldots\ldots\ldots$$

If cash flows are distributed to share holders as dividends $D_t = P.F(K_t)$, then the value of a company is equal to the discounted value of future dividends.

$$V_0 = \frac{D_{t+1}}{(1+r)} + \frac{D_{t+2}}{(1+r)^2} + \frac{D_{t+3}}{(1+r)^3} + \ldots\ldots\ldots\ldots$$

If a firm decides not to pay a dividend, but invests the proceeds in an interest bearing bond then it is reducing current dividends, but increasing the future cash flows earned by the firm. Hence:

$D_{t+1} = 0$, but $D_{t+2} = (1+r)D_{t+1} + P.F(K)$

Substituting this into the valuation of a firm, we can see that there has been no change to the present discounted value of dividend payments. In this case, there is no difference in the share prices of the two firms.

However, if firm two were to make a one period investment in an asset that yields a rate of return higher than the interest rate its share price would rise relative to firm one, reflecting the increase in the discounted value of future dividend payments. But conversely, if firm two invests in an asset or a project that yields a lower rate of return than the interest rate its share price would fall relatively.

8. What are the advantages and disadvantages for a firm in holding inventories? What factors might determine the size of the inventory stock that firms decide to hold? What factors determine the sensitivity of inventory investment to output?

The advantages of holding inventories:
Avoid stock outs- the situation where a firm is unable to fulfil unexpected or unplanned orders
Smooth production- if production experiences strong diminishing returns at higher levels of output, smoothing production enables lower average costs in the long run.

The disadvantages of holding inventories
Storage costs- holding inventories requires adequate storage facilities.
The cost of holding inventories includes depreciation, i.e. that inventory stocks might degrade or be superseded by higher technology products
There is difficulty in predicting demand changes, which limits the potential success of an inventory policy.

It is assumed that firms operate an optimal inventory-output ratio $v = \frac{K_{INV}}{Y}$ where K_{INV} is the stock of inventories. It is assumed that the ratio v rises as the benefits of holding inventories rise relative to the costs.

Where the firm production function exhibits strong diminishing returns (concavity) the benefits of production smoothing are greater, hence an inventory policy is likely to be used to meet fluctuations in demand. However, if the cost of storing inventories including possible depreciation is high optimal inventory stocks to output would be low.

The parameter *v* will also determine the sensitivity of inventory investment to output. Where there are high inventory stocks, changes in output can be met by adjusting the stock of inventories, hence inventory investment would be less sensitive to demand movements. In times of high demand the inventory stock would simply run down, whereas in times of low demand inventory stocks would be naturally replenished. However, when inventory stocks are low inventory investment is much more sensitive to output movements.

Hence, a plausible model of inventory investment could take the following form:

$$I_{INV} = \Delta K_{INV} = \frac{1}{v}\Delta Y$$

9. Explain how the following might affect residential investment:

a. increased availability of mortgage credit

A q- theory of residential investment can take a similar form to that applied to business investment. The value of a residential investment, V_0^H , should be determined by the discounted value of the future rents $\left(R_1, R_2, R_3, ...\right)$ that it will earn.

$$V_0^H = \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + ...$$

If the rent is the same in all periods, $R = R_1 = R_2 = R_3 = \ldots$, then this will simplify to:

$$V_0^H = \frac{R}{r}$$

If the physical cost of building a residential unit is given by the construction price, P_H then a q-theory for residential investment $\left(I_H\right)$ would take the following form:

$$q_H = \frac{R}{rP_H}$$

$$I_H = \frac{1}{\theta_H}\left(q_H - 1\right)$$

An increase in the availability of mortgages would be expected to increase the demand for houses and then bid up house prices and rents. Because the flow of rents from a unit of residential investment are now greater, q_H will rise above unity stimulating investment.

b. an increase in the supply of land for development

This would be expected to lower the price of land, and hence the cost of a residential unit P_H. Consequently, the ratio of discounted rental payments to the cost of residential capital will rise, increasing both q_H and raising investment.

c. new legislation that increases the time for planning decisions to be taken

The parameter θ_H represents the costs involved in adjusting the stock of residential capital. As this parameter increases, the speed at which investment responds to a deviation of q_H from unity falls. Therefore, an increase in planning time wouldn't be expected to have any effect on the determinants of q_H, but by raising adjustment costs is likely to make residential investment less sensitive to any movement in q_H.

More advanced problems

10. A firm faces a production function of the form $Y = AK^{\alpha}$, where Y is output, A is the level of productivity, K is the installed capital stock

a. What is the marginal product of capital?

The marginal product of capital (*MPK*) is the derivative of the firm production function $Y = AK^{\alpha}$ with respect to *K*.

$MPK = \alpha AK^{\alpha-1}$

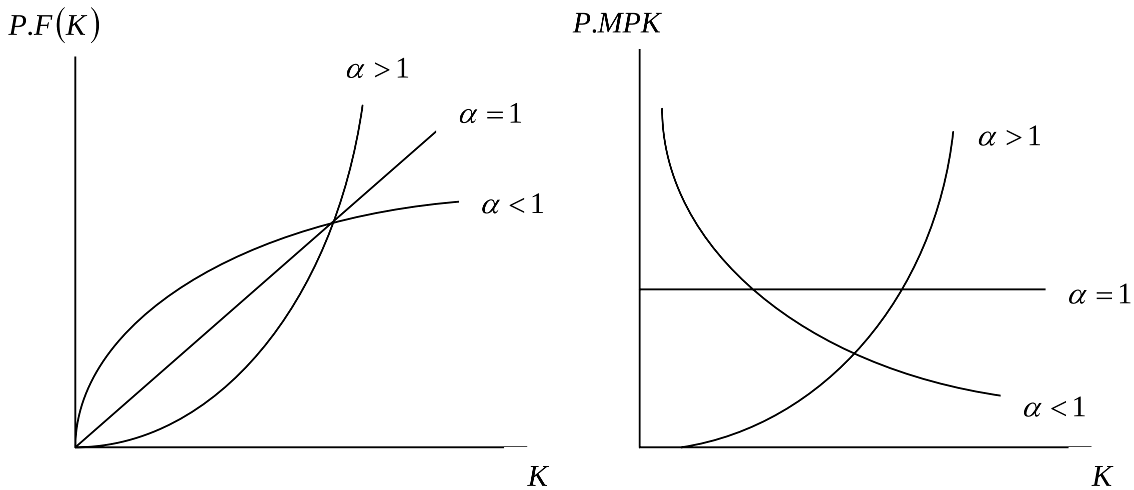
It simply informs of the change in total output when the capital stock varies by one unit.

b. The firm sells its output at price P. Calculate and sketch the firm's revenue and marginal revenue product functions.

The firm's revenue (*R*) is the product of total output and the average price of output. $R = P.Y = P.AK^{\alpha}$. The marginal revenue product of capital (MRPK) is the product of the MPK and prices, and measures how much total revenue changes when the capital stock varies by one unit.

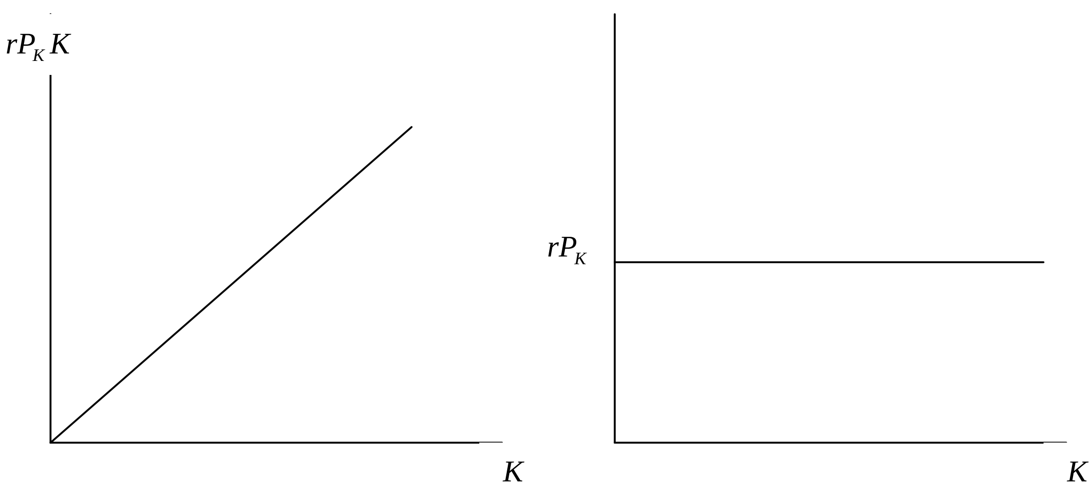
$MRPK = P.\alpha AK^{\alpha-1}$

The firms' revenue and marginal revenue product depends on the value of α in the production function. When $\alpha < 1$ the production function exhibits diminishing returns with respect to capital, hence the revenue function is concave and the marginal revenue product of capital is downward sloping. If $\alpha = 1$ then firm production has constant returns with respect to capital, the revenue function is linear and the marginal revenue product of capital is constant at all levels of capital stock. Finally, if $\alpha > 1$ there are increasing returns to capital. Both the revenue and marginal revenue products are convex showing that the marginal returns to capital increase as the capital stock grows.



c. The firm can rent each unit of capital from a capital leasing firm. The purchase price is P_K and the rental rate r. Derive and show the firm's total and marginal cost of capital.

The total costs of production are linear with respect to capital $C = rP_K K$, with the marginal cost of capital equal to rP_K.



d. Calculate the optimal capital stock of the firm, and show how it is affected by a change in any of the parameters mentioned above.

The optimal capital stock K^* is determined by the firm's profit maximising position.

$$\Pi = AK^{\alpha} - rP_K K$$

$$P.\alpha AK^{\alpha-1} - rP_K = 0$$

Profits are maximised where the marginal cost of the capital stock are equal to its marginal revenue product. Rearranging and simplifying the above gives:

$$K^* = \left(\alpha A \frac{P}{rP_K}\right)^{\frac{1}{1-\alpha}}$$

Note, that this only has a closed form solution if $\alpha < 1$. If there are increasing returns to scale then naturally $K^* \to \infty$. With constant returns to scale $K^* = \infty$ if $P.A > rP_K$ or $K^* = 0$ if $P.A < rP_K$. It is normal to assume that production exhibits diminishing returns to each factor of production.
It can be seen that the optimal capital stock will respond to the parameters in the model in the following way:

Productivity (A): $\frac{\Delta K^*}{\Delta A} > 0$

Market price (P): $\frac{\Delta K^*}{\Delta P} > 0$

Capital goods price (P_K): $\frac{\Delta K^*}{\Delta P_K} < 0$

Interest rates: $\frac{\Delta K^*}{\Delta r} < 0$

11. Explain how the existence of credit constraints might affect investment in the Tobin's q model. What factors might lead to financial markets imposing credit constraints?

From the q model of investment, there will be upward pressure on investment when $P.F(K) > rP_K K$. In this scenario, the increase in firm valuation from an increase in capital stock exceeds the cost of installing that capital.

However, as investment expands the value of q will return to unity as $P.F(K) = rP_K K$ is restored. This is because the right hand side of this equation is assumed to exhibit diminishing returns to capital and a falling marginal product of capital, whereas there is a constant marginal cost of capital. Therefore, as the capital stock expands equality is restored.

However, suppose there are binding borrowing constraints. In this case q may get stuck at a value greater than unity. This is because the necessary expansion in the capital stock through extra investment that is warranted cannot be undertaken due to a lack of finance. Therefore, even if q-theory indicates a change in investment, that investment may fail to occur if there are binding credit constraints.