Chapter 11: Probability

Extra questions

Basic Probability:

1. A large group of people consists of equal numbers of men and women. If an individual is chosen at random, what is the probability of selecting a woman?

2. From the same group, if two individuals were selected at random and with replacement, what is the probability of them both being men?

3. Explain the importance of the phrases “at random” and “with replacement” in Question 2.

4. If two six-sided dice are thrown, what is the probability of an odd total score?

5. What is the probability of selecting two Spades when selecting two cards from a standard pack (a) with replacement and (b) without replacement?

6. What is the probability of selecting an Ace or a Heart on a single selection from a normal pack of cards?

7. A group of eight friends are going on holiday together. There are five girls and three boys travelling in the people carrier that they have borrowed. If Customs select one person at random to question, what is the probability of it being one of the boys? A second Customs officer arrives, and not realising that one person has already been selected to be questioned, selects another one. What is the probability that the second person is one of the boys? What is the probability that both people selected were girls?

Conditional Probability

8. Two people work in a shop and both have very few complaints about their work from customers. In fact, 90% of customers rate both Adele and Franz as “Good”. Ten percent of customers served by Adele have minor complaints, as do 9% of those served by Franz. One percent of Franz’s customers have written to praise his work.
   (a) If a customer is chosen at random and rates the service as “God”, what is the probability they were served by Adele?
   (b) If a letter of praise was sent, what is the probability it was for Franz?
   (c) If a customer has complained, what is the probability they were served by Adele?

9. Evaluate the following term:

$$\binom{12}{3}$$

10. In a very large group of students, 80% pass a test. If a group of five is selected at random, what is the probability:
    (a) all passed
    (b) four passed
    (c) at least three passed
    (d) what assumptions have you made in coming to your answers?

Extra Answers

1. If there are equal numbers of men and women, then there must be an equal chance of selecting a man or a woman. Equal chance means equal probability, so the answer must be $^1/_2$.

2. Since it is with replacement, the probability remains the same at $^1/_2$. So the probability of both being men is $^1/_2 * ^1/_2 = ^1/_4$

3. "at random" suggests that there is no human interference in the selection and this should ensure no bias in selection.
"with replacement" means that the probabilities remain constant. IN a small group, probabilities can change quite radically if people are selected without replacement.

4. There are 36 outcomes in the sample space, but not 36 different total scores. It may be simplest to draw a picture of the sample space and count up the number of odd scores.

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

So there are 18 odd scores out of 36 outcomes, so the probability is 1/2

5. (a) In this case the probability remains the same from the first event to the second, so we have independent events. The probability is $^1/_4 * ^1/_4$ = 1/16
(b) without replacement the events are dependent – the second probability depends on the outcome of the first event. Since we want 2 Spades, we have $^1/_4$ * 12/51 = 12/204 = 0.0588235

6. Here the events are not mutually exclusive, since the 13 Hearts include the Ace, and the 4 Aces include a Heart. Using the standard rule, we have P(Ace) + P(Heart) – P(Ace of Hearts)
4/52 + 13/52 – 1/52 = 16/52
Or you could think of it as 13 Hearts and 3 other Aces, making 16 out of 52

7. There are three boys out of eight, so the P(boy) = 3/8
Now there are only two boys, so P(boy) = 2/7
For two girls we have (5/8)*(4/7) = 5/14

Conditional Probability:

8. (a) Since the level of complaint is the same for both people, the probability must be the same as the proportion served, here 0.6
(b) Since letter only come about Franz, the probability must be 1
(c) the probability of the server being Adele and there being a complaint is 0.6*0.1 = 0.06
the probability of a complaint is 0.6*0.1+0.4*0.09 = 0.06+0.036 = 0.096
So the probability it was Adele is 0.06/0.096 = 0.625 or 5/8

9. Here we can use the standard formula:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{12!}{3!(12-3)!} = \frac{12!}{3!9!} = \frac{12.11.10.9!}{3.2.9!} = \frac{12.11.10}{3.2} = \frac{1320}{6} = 220$$

10. This is a binomial problem, with p = 0.8 and n = 5

(e) All pass is $\binom{5}{5} p^5 q^0 = 1 \times (0.8)^5 \times 1 = \mathbf{0.32768}$

(f) Four pass is $\binom{5}{4} p^4 q^1 = 5 \times (0.8)^4 (0.2)^1 = 5 \times \mathbf{0.4096} \times \mathbf{0.2} = \mathbf{0.4096}$

(g) At least three pass is 3 or 4 or 5 pass, so we add the probabilities. We have already worked out two of these answers, so only need to work out P(3 pass) and add this to the other answers

$\binom{5}{3} p^3 q^2 = 10 \times (.8)^3 (.2)^2 = 10 \times \mathbf{0.512} \times \mathbf{0.04} = \mathbf{0.2048}$

Adding the three answers gives the probability that at least three of the group passed as 0.94208

(h) we have assumed that the group is so large that the probability does not change when we remove or select a person.