

## Chapter 3: Using Graphs

### Extra questions

#### Time Series:

1. For the following annual data, construct a time series graph:

Year	Sales	Year	Sales
1	120	6	185
2	130	7	204
3	140	8	196
4	155	9	190
5	168	10	191

2. The following data was collected at monthly intervals. Construct a graph for this data:

Year	1	2	3	4
January	25	35	46	58
February	20	22	36	44
March	30	36	43	51
April	50	60	66	72
May	70	90	90	95
June	120	130	150	140
July	190	210	230	240
August	250	280	290	300
September	170	178	194	193
October	120	130	135	133
November	60	75	83	77
December	20	25	29	26

#### Break-even Analysis:

3. A small company makes coats which it sells at 20 euros. It has a fixed cost of 1,000 euros per month, and the labour and material cost of a coat is 9 euros. What is the break-even point for the company?
4. Sean has had an idea for a business. He wants to write web pages for small companies who cannot afford to employ expensive web designers and plans to charge only £2 per page. He is still living at home so all of his costs can be considered as fixed – he pays his parents a small rent of £50 per week, but doesn't pay for electricity or anything else separately. He estimates the cost of getting a job is £25, this includes travel, mock-up pages and such things. An average site for such businesses would have about 15 pages. What is his break-even level of work?
5. A business is trying to decide on renting some premises. They are faced by a choice of various sites, all of which offer full service for a single monthly fee. Site A costs £1,000 per month, Site B £1,200 per month and Site C £1,500 per month. The contribution from the single product produced by the firm is £15. Advise the firm on the break-even levels for each site.

#### Linear Programming:

6. A company produces hand crafted models in two types, X and Y. Each X takes 6 hours of labour, whilst each Y only takes 5 hours. The models are made from clay, X's taking 3 kilo's and Y's taking a single kilo. A local shop

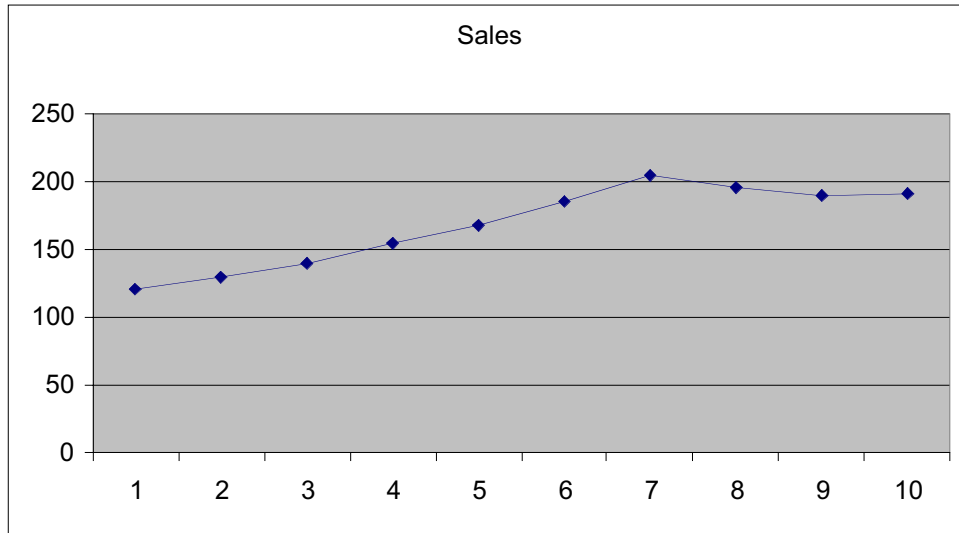
takes at least 5 Y's per week and therefore the company always produces 5 or more Y's per week. Total labour time available is 300 hours per week. Total clay available is 120 kilo's per week. The company tells you that the contribution to profit from one X model is £3 and the contribution from a Y model is £5. Formulate this as a linear programming problem; construct a graph showing the feasible area and hence advise the company on the numbers of X and Y it should produce each week to maximise profits.

7. The Movement Company wishes to minimise the costs of journeys undertaken by its lorries. The Humber lorry can carry 6 palettes of the product Firstly or 5 palettes of product Andalso. The Dragall lorry can carry one palette of Firstlys or 8 palettes of Andalsos. The company needs to move a minimum of 120 palettes of Firstlys and 400 palettes of Andalsos per month. For technical reasons, a minimum of 20 journeys per month must be made by the Dragalls. The cost of a journey by Humber is £5 and by Dragall is £6. Advise the company on the number on the minimum number of journeys required by each type of lorry.

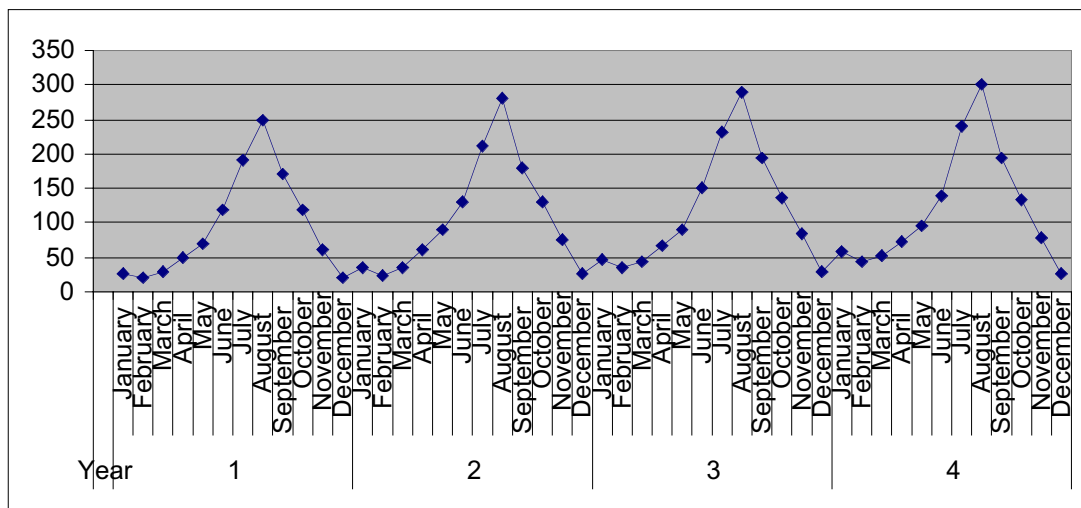
## Extra answers

### Time Series:

1. Your graph should look like this if you used Excel:



2. Using Excel gives:



### Break-even Analysis:

3. Firstly we need to work out the contribution from a coat. This is Price minus cost:  $20 - 9 = 11$  euros. We now divide the fixed cost per month by the contribution:  $1,000/11 = 90.909$ . Since we are concerned with complete coats, we would say that the **break-even level for the company is 91 coats per month**. Just to confirm this, the costs at this production level are  $1,000 + 91 \times 9 = 1,819$  euros. The revenue is  $91 \times 20 = 1,820$  euros; the difference being accounted for by the rounding up of the break-even figure.
4. The fixed cost is £50 per week, which he pays to his parents. To find the revenues, we take the price per page (£2) and multiply by the average number of pages (15) to get a likely revenue per job of £30. If the cost of getting a job is £25, then the contribution is  $£30 - £25 = £5$ . We now divide the fixed cost by the contribution ( $50/5$ ) to get the number of jobs needed per week. **The answer is 10 per week**. (You might question if this is viable long term, it is a lot of web pages to write every week (150), and he will need to

make allowances for new computers and software as time passes.)

5. In each case we need to divide the monthly fee by the contribution. **For Site A this gives 666.67; for Site B 80; and for Site C 100.** You would need to know the firm's monthly sales, probably over a relatively long period, in order to advise them on which site to choose. There would, of course, be many other factors to take into account before a site was chosen.

**Linear Programming:**

6. The wording of this question makes it easier than it might be. Firstly we look at labour time – 300 hours in total, X takes 6 and Y takes 5, so we get:

$$6X + 5Y \leq 300$$

Now take clay, total is 120, X takes 3, Y takes 1, so:

$$3X + Y \leq 120$$

The shop takes 5 Y per week, so we must have  $Y \geq 5$

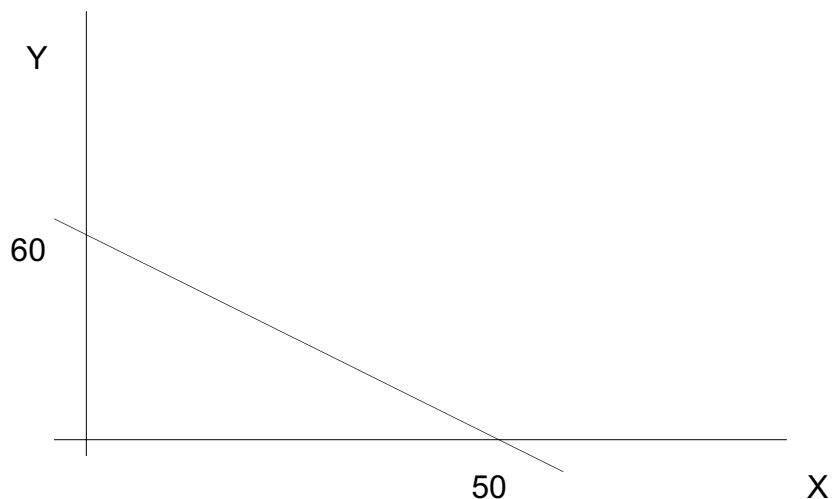
Finally the profit contribution is 3 from X and 5 from Y, so:

$$\text{Profit} = 3X + 5Y$$

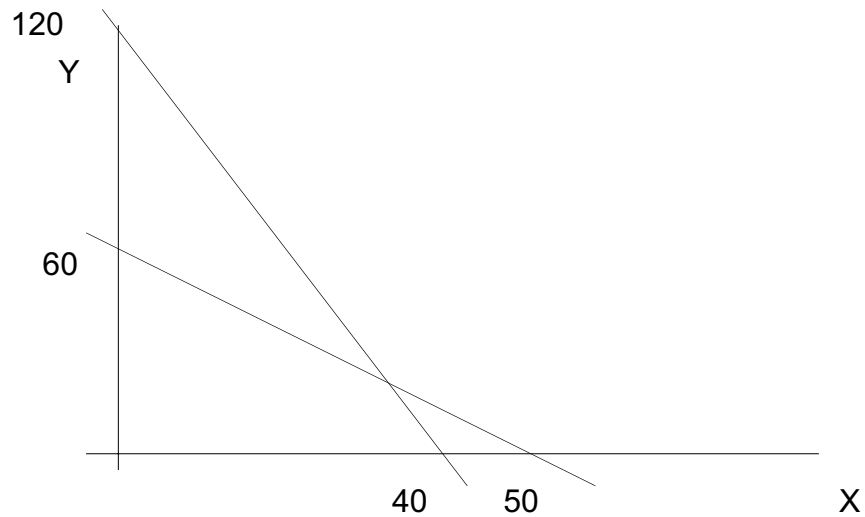
To complete the formulation stage, we note that  $X \geq 0$

**The Graph:**

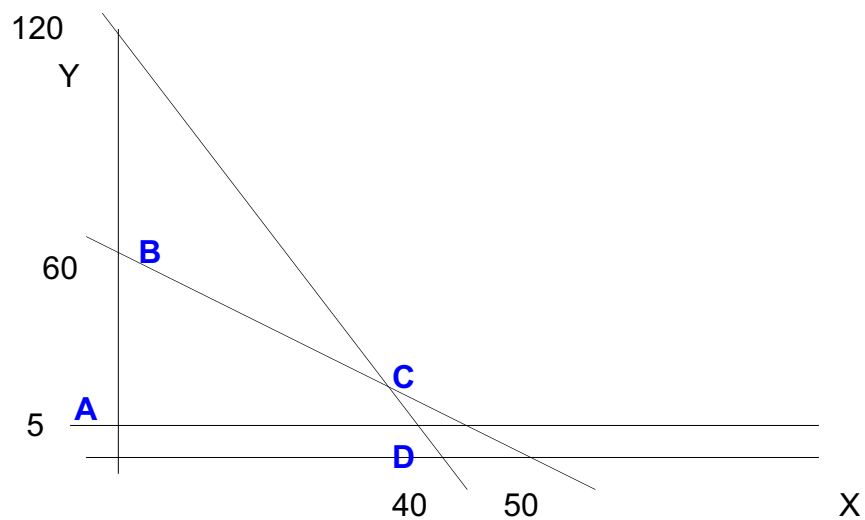
Taking the labour constraint, if  $Y = 0$ , then  $X = 50$ ; if  $X = 0$ , then  $Y = 60$ . This now gives us the following graph:



Taking the clay constraint, if  $Y = 0$ , then  $X = 40$ ; if  $X = 0$ , then  $Y = 120$ . Adding this to the graph, we have:



Finally we can add the  $Y \geq 5$  constraint to get:



We have labelled the corners of the feasible area A, B, C, D and we must now find the X and Y values for these. We could just read them from the graph, but normally they are calculated. Point A is (0,5) and point B is (0,60). Point C we can find either from the graph or by solving the simultaneous equations. In either case we get (33.33, 20). Point D we find by substituting  $Y = 5$  into the Clay constraint, to get (38.333, 5).

If we now take each of these corners and put the X and Y values into the Profit function, we get:

A	B	C	D
(0, 5)	(0, 60)	(33.33, 20)	(38.33, 5)
15	300	200	140

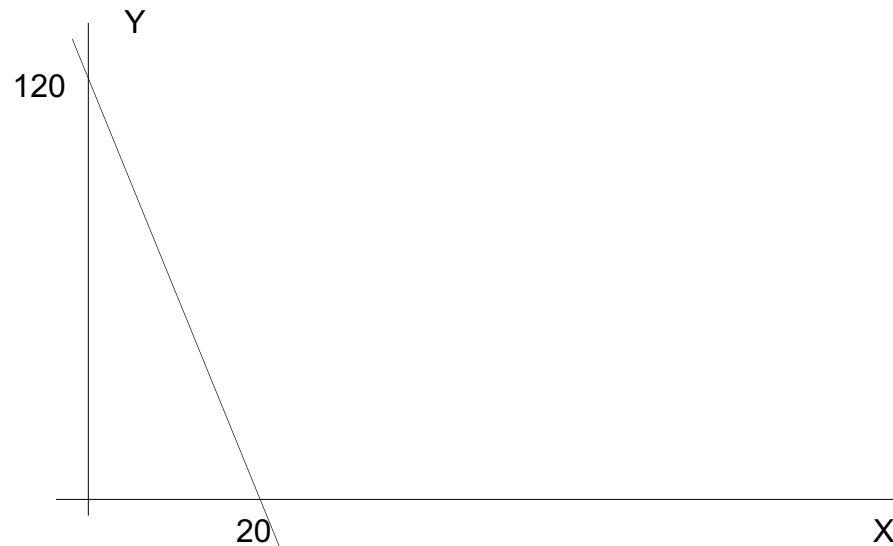
**So maximum profit is achieved at Point B and the company should produce 60 Y's and no X's.**

7. This is a minimisation problem. The first step might be to specify the cost function. If we let X represent Humber lorries and Y represent Dragall lorries, we have:

$$\text{Costs} = 5X + 6Y$$

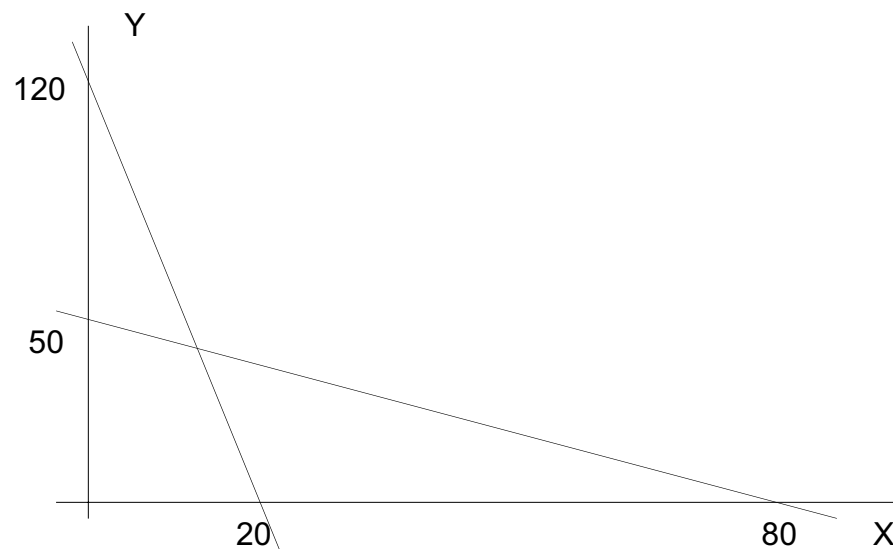
Looking at the product Firstlys, the company want to move at least 120 palettes per month. Humpers carry 6 and Dragalls carry one, so:

$$6X + Y \geq 120$$



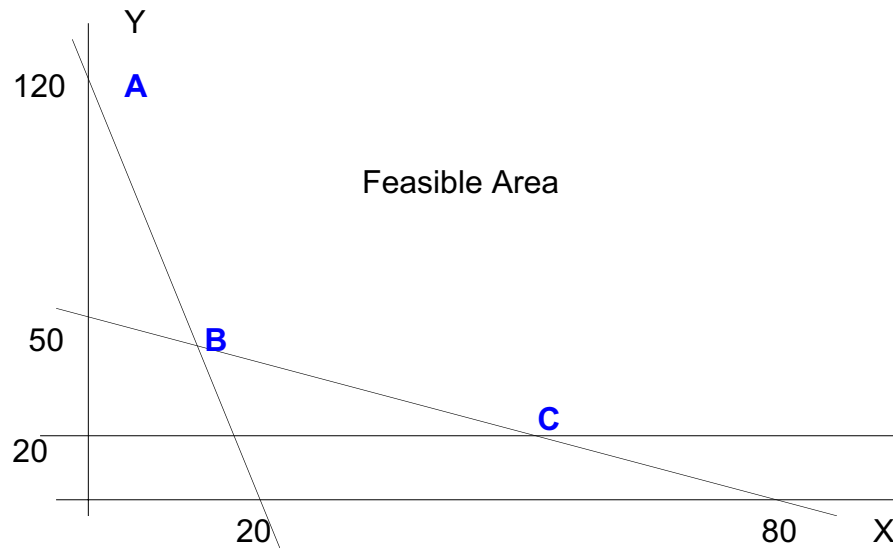
Now for Andalsos, they need to transport a minimum of 400 palettes and we know that Humpers carry 5 whilst Dragalls carry 8, giving:

$$5X + 8Y \geq 400$$



Finally, the minimum level of 20 journey by the Dragalls is represented by:

$$Y \geq 20$$



The three corners of the feasible space are labelled A, B, and C – remember that we are trying to get as near as possible to the origin. We can put the values into the Cost function to get:

A	B	C
(0, 120)	(13.02, 41.86)	(48, 20)
720	316.26	360

So the minimum cost corner is B and we recommend an average of 13 trips by Humper and 41.86 trips by Dragall