CHAPTER 17

Theory of Computation

(Solutions to Odd-Numbered Problems)

Review Questions

1. The three statements in our Simple Language are the increment statement, decrement statement, and loop statement. The increment statement adds 1 to the variable; the decrement statement subtracts 1 from the variable; the loop statement repeats an action (or a series of actions) while the value of the variable is not zero.

3. A problem that can be solved by our Simple Language can also be solved by the Turing machine.

5. One way to delimit the data on a Turing machine tape is the use of two blanks, one at the beginning of the data and one at the end of the data.

7. A transition state diagram is a pictorial representation of a program written for the Turing machine.

9. A Gödel number is an unsigned integer that is assigned to every program that can be written in a specific language. In the halting program, we represent a program as its Gödel number when that program is the input to another program.

Multiple-Choice Questions

11. a 13. b 15. a 17. c 19. d 21. c
23. c 25. c 27. d

Exercises

29. See Algorithm S17.29. After assigning Y to Z, we increment Z (X times).
Algorithm S17.29  Exercise 29

\[
\text{Temp} \leftarrow X \quad \text{// See solution to Exercise 28}
\]
\[
Z \leftarrow Y \quad \text{// See solution to Exercise 28}
\]
\[
\text{while (Temp)}
\]
\[
\quad \text{decr (Temp)}
\]
\[
\quad \text{incr (Z)}
\]

31. See Algorithm S17.31.

Algorithm S17.31  Exercise 31

\[
\text{Temp} \leftarrow X \quad \text{// See solution to Exercise 28}
\]
\[
Z \leftarrow 1 \quad \text{// See algorithm 17.8 in the text}
\]
\[
\text{while (Temp)}
\]
\[
\quad \text{decr (Temp)}
\]
\[
\quad Z \leftarrow Z \times Y
\]

33. See Algorithm S17.33.

Algorithm S17.33  Exercise 33

\[
\text{Temp} \leftarrow X + 1
\]
\[
\text{while (X)}
\]
\[
\quad \text{decr (X)}
\]
\[
\Lambda_1
\]
\[
\text{Temp} \leftarrow 0
\]
\[
\text{while (Temp)}
\]
\[
\quad \text{decr (Temp)}
\]
\[
\Lambda_2
\]

35. The tape moves to the right and goes to state B as shown below:

\[
\downarrow
\]

\[
\begin{array}{cccccc}
\text{b} & \text{l} & \text{l} & \text{l} & \text{b}
\end{array}
\]

State: B

37. Figure S17.37 shows the state diagram.
Figure S17.37  Exercise 37

![State Diagram]

S₁ : starting  S₂ : moving right  S₃ : moving left  S₄ : take care of carry  S₅ : add  S₆ : halt

39.

Start

\[ \ldots \ b \ b \ b \ b \ldots \]

State: S₁

\[ X = 0 \]

(S₁, b, b, R, S₂)

\[ \ldots \ b \ b \ b \ b \ldots \]

State: S₂

\[ X = 0 \]

(S₂, b, b, N, S₃)

\[ \ldots \ b \ b \ b \ b \ldots \]

State: S₃ (halt)

\[ X = 0 \]

41.

a. (S₁, b, b, R, S₂) — S₁ is the starting state.

b. (S₂, 1, 1, R, S₂) — S₂ is the move right state.

c. (S₂, b, b, L, S₃)

d. (S₃, 1, b, L, S₃) — S₃ is the move left state. 1 is changed to b.

e. (S₃, b, b, N, S₄) — S₄ is the halt state.

43. We use a single 1 to represent 0, two 1’s to represent 1, three 1’s to represent 2, …, and \( n + 1 \) 1’s to represent \( n \).

45. Algorithm S17.45 shows the statements for the macro and the Gödel number for each statement.

Algorithm S17.45  Exercise 45

\[
\begin{align*}
X₂ & \leftarrow 0 & \text{// Gödel Number: } & \text{CF2DBF2E} \\
\text{inc}r \ X₂ & \text{ // Gödel Number: } & \text{AF2} \\
\text{inc}r \ X₂ & \text{ // Gödel Number: } & \text{AF2}
\end{align*}
\]

The Gödel number for the macro is then \((\text{CF2DBF2EAF2AF2})_{16}\). Notice that this micro does not preserve the value of \( X₂ \). The Gödel number for the macro will be longer if we want to preserve \( X₂ \).