CHAPTER 3

CONCEPT REVIEW QUESTIONS

1. Will a deposit made into an account paying *compound interest* (assuming compounding occurs once per year) yield a higher future value after one period than an equal-sized deposit in an account paying *simple interest*? What about future values for investments held longer than one period?

Compounded interest returns more than simple interest over multiple periods because you earn interest on the interest. For the very first period, however, the return is the same with simple or compound interest. Holding an investment longer will also mean a higher future value, because there is more time for the investment to earn interest.

2. How would (a) a *decrease* in the interest rate or (b) an *increase* in the holding period of a deposit affect its future value? Why?

A decrease in the interest rate would lower future value, while an increase in the holding period will increase future value. Decreasing the interest rate decreases the future value factor and thus future value. Increasing the holding period increases the future value factor and thus future value.

3. What would happen if someone had a negative rate of interest paid?

This person would incur a net loss.

4. How are the present value and the future value of a lump sum related—in definition and in terms of mathematics? Notice that for a given interest rate (*r*) and a given investment time horizon (*n*), $PVF_{r,n}$ and $FVF_{r,n}$ are inverses of each other. Why?

Present value and future value are related because the future value of an amount of money is equal to its present value times one plus the interest rate raised to the nth power, the number of years money is being compounded. Similarly, present value is future value divided by one plus the interest rate raised to the nth power. This means, given present value, interest and number of years you can find the future value amount that is equivalent to this sum. Present value and future value are inverses of each other because this equation is finding equivalent amounts – what present value equates to a future value at a specified number of years at a certain interest rate.

5. How would (a) an *increase* in the discount rate or (b) a *decrease* in the time period until the cash flow is received affect the present value? Why?

An increase in the discount rate decreases the present value factor and the present value. This is because a higher interest rate means you would have to set less aside today to earn a specified amount in the future. A decrease in the time period increases the present value factor and increases the present value. This is because if you have less time, you will have to set aside more today to earn a specified amount in the future.

6. How would *the future value of a mixed stream of cash flows* be calculated, given the cash flows and applicable interest rate?

The future value of a mixed stream of cash flows would be calculated by taking each individual cash flow, and then compounding it, using future value factors or

the future value formula to take that amount into the future. Then add up all of the future-valued amounts for the final future value of the mixed stream of cash flows.

7. Differentiate between an *ordinary annuity* and an *annuity due.* How is the future value of an ordinary annuity calculated, and how (for the same cash flows) can it be converted into the future value of an annuity due?

In an ordinary annuity, payments are received at the end of the period. Payments are received or made at the beginning of the period in an annuity due. Cash flows can be converted from an ordinary annuity into an annuity due by multiplying the final answer by one plus the interest rate.

8. What would happen to the relationship between the values of annuities due and ordinary annuities if interest rates were to be negative?

Cash flows could be converted from an ordinary annuity into an annuity due by multiplying the final answer by one minus the interest rate.

9. How would the *present value of a mixed stream of cash flows* be calculated, given the cash flows and an applicable required return?

You could find the present value of a mixed stream of cash flows using the cash flow menu of a financial calculator or you could individually discount each cash flow to the present using present value factors or the present value formula and then sum all of the values.

10. Given the present value of an *ordinary annuity* and the applicable required return, how can this value be easily converted into the present value of an otherwise identical *annuity due?* What is the fundamental difference between the cash flow streams of these two annuities?

An ordinary annuity can easily be converted into an annuity due by multiplying the ordinary annuity value by one plus the interest rate. The main difference is that you are receiving (or paying) one more payment with an annuity due than with an ordinary annuity since the payments are made at the beginning of the month instead of the end of the month.

11. What is a *perpetuity,* and how is its present value conveniently calculated? How do you find the present value of a *growing perpetuity*?

The value of a perpetuity can be easily calculated by dividing the perpetual cash flow by the interest rate. The value of a growing perpetuity is the cash flow in period 1 divided by the interest rate minus the growth rate.

12. What if we have a mixed stream of cashflows that also has a mixed set of interest rates? Can we handle this using the processes above?

Yes. Each cash flow can be discounted or compounded separately at the appropriate rate, with the PVs or FVs added up to find the final answer.

13. What effect does increasing compounding frequency have on the (a) future value of a given deposit and (b) its *effective annual rate (EAR)?*

Increasing the compounding period increases the future value of a given deposit and increases the effective annual rate (EAR).

14. Under what condition would the stated annual rate equal the effective annual rate (*EAR*) for a given deposit? How do these rates relate to the *annual percentage rate (APR)* and *annual percentage yield (APY)?*

The stated annual rate is equal to the EAR when there is annual compounding. The annual percentage rate (APR) is the non-compounded rate, in other words the annual rate when there is no compounding. The annual percentage yield (APY) is the annual return on an investment.

15. How would you determine the size of the annual end-of-year deposits needed to accumulate a given future sum, at the end of a specified future period? What effect does the magnitude of the interest rate have on the size of the deposits needed?

To find the annual deposits needed to accumulate a given sum, you would need to solve an annuity problem, using the annuity formula, annuity factor tables or a financial calculator. The future value (FV) would be given in the problem, along with the interest rate (I) and number of years (N). You can then solve for payment required, PMT. The higher the interest rate, the lower the payment required

16. What relationship exists between the calculation of the present value of an annuity and amortization of a loan? How can you find the amount of interest paid each year under an amortized loan?

An amortized loan is an annuity with a repayment of interest and principal in each payment. You are computing a new loan balance after each payment is made, and therefore a new amount of interest and principal repaid. You can find the amount of interest paid by setting up a table which subtracts the amount of principal paid with each payment from the balance. You can recompute interest on the new balance. Any part of the payment that is not interest expense is the principal repayment.

17. How can you find the interest or growth rate for (a) a lump sum amount, (b) an annuity, and (c) a mixed stream?

The interest rate for all three can be found if we know all the other variables in the FV formula by using a financial calculator or excel spreadsheet.

a. $FV = PV \times (1+r)$

b. $FV = PMT \times \left[\frac{(1+r)^n - 1}{r} \right]$

c. $FV = \sum_{t=1}^{n} CP_t \times (1+r)^{n-t}$

18. How can you find the number of time periods needed to repay (a) a single-payment loan and (b) an installment loan requiring equal annual end-of-year payments?

a. If the present (PV) and future (FV) amounts are known along with the interest rate (r), we can calculate the number of periods (n) necessary for the present amount of the loan to grow to equal the future amount.
b. Using a financial calculator or excel spread sheet, if we know the amount of the loan (PV), the amount of the payments and the interest rate (r), using the formula for FV of an ordinary annuity.